Quantum decoherence, Zeno process, and time symmetry breaking

T. Petrosky and V. Barsegov

Center for Studies in Statistical Mechanics and Complex Systems, The University of Texas at Austin, Austin, Texas 78712

The complex spectral representation of the Liouville–von Neumann operator outside Hilbert space is applied to the decoherence problem in quantum Brownian motion. In contrast to the path-integral method, often used in the context of quantum decoherence for the case where the environment surrounding the Brownian particle (subsystem) is in thermal equilibrium, our spectral representation is applicable to systems far from equilibrium, including a pure state for the surrounding bath. Starting with this pure initial condition, the subsystem evolves in time obeying a diffusion-type kinetic equation. Hence, the collapse of wave functions is a dynamical phenomenon occurring outside Hilbert space, and is not simply a contamination of the subsystem, a popular view accepted in the so-called "environmental" approach, by the mixed nature of the thermal bath. The essential element in the understanding of quantum decoherence is the "extensivity" of quantities characterizing the thermodynamic limit. Quantum Zeno time is shown to be a lower bound of the decoherence time.

I. INTRODUCTION

In this paper, we shall apply the *complex spectral representations* of the Liouville–von Neumann operator (the Liouvillian in short) for the density matrix to the quantum decoherence problem.

The recent development of the complex spectral representations shows that irreversibility is a rigorous dynamical process taking place outside Hilbert space in the thermodynamic limit (see [1] and references therein for deterministic maps, [2–4] for decaying states in quantum mechanics, and [5–7] for thermodynamical systems).

A popular trend is to attribute irreversibility to quantum decoherence [8,9]. This involves a basic distinction between the open system and its environment, which is assumed to be in a thermodynamic equilibrium. This distinction introduces an anthropomorphic element. Indeed, the environment introduced by one observer, may be different to that seen by another observer. Moreover, this distinction is phenomenological as one avoids answering the most fundamental question of nonequilibrium statistical mechanics, namely, what is the mechanism that governs the approach to thermal equilibrium of the environment, without violating the basic laws of physics.

The problem of irreversibility is especially important in quantum mechanics, which has been remarkably successful in all its predictions. Still discussions about its meaning and scope are as lively as ever. The basic assumption in quantum mechanics is that every problem can be solved at the level of wave functions, that is, probability amplitudes. This leads, however, to the well-known duality between the Schrödinger equation, which is time reversible and deterministic, and the reduction or collapse of the wave function, which evolves from a single wave function to a *mixture* described by a density matrix. This duality seems to imply that we need our measurements in order to go from "potentialities" to "actualities" [10]. For Bohr and Rosenfeld [11], measurements are irreversible processes, be it at the level of the apparatus or at the level of our sensory mechanisms.

The need to go outside Hilbert space in quantum mechanics for the case of a continuous spectrum was recognized some years ago by various physicists and mathematicians including Sudarshan, Chiu, and Gorini [2], Böhm and Gadella [3,12], and Kumicak and Brändas [13]. The physical motivation was to include decaying states as observed in the spectral decomposition of the Hamiltonian (hence, the name *Gamow vectors* used by Böhm and Gadella [12]). However, this generalization, while going in the right direction, does not solve the "quantum paradox" associated with the duality mentioned above. To solve these problems we have to turn to the Liouville space (i.e., the density matrix space) and show that in the cases where we expect dissipation, we obtain new spectral decompositions that lead to *semigroups* including irreversible processes [6].

The complex spectral representation of L_H is quite remarkable, as it exhibits "non-Schrödinger" features in quantum mechanics. Indeed, there appear dissipative effects associated with collision operators of the Pauli type familiar from phenomenological kinetic theories. Our method, therefore, leads to a unification of dynamics, thermodynamics, and kinetic theory.

The complex spectral representation leads to a new concept, *subdynamics*, which decomposes the dynamical evolution into independent components [14–16]. This extension becomes essential in the case when memory effects are nonintegrable. For example, this is the case whenever one goes beyond weakly coupled systems. Indeed, since the original work of Prigogine and Résibois [17,18], it is well-known that the Liouville–von Neumann equation lead to the generalized master equation for the reduced distribution function $P^{(\nu_1)}\rho(t)$ as

$$i\frac{\partial}{\partial t}P^{(\nu_1)}\rho(t) = P^{(\nu_1)}L_0 P^{(\nu_1)}\rho(t)$$

$$+ \int_0^t dt'\, \delta\tilde{\psi}^{(\nu_1)}(t')P^{(\nu_1)}\rho(t-t'). \quad (1)$$

Here, $P^{(\nu_1)}$ is a projection operator defined in Eq. (35); L_0 is the unperturbed Liouvillian [see Eq. (17)]; $\delta\tilde{\psi}^{(\nu_1)}(t)$ is the

inverse Laplace transformation of the collision operator $\delta\psi^{(\nu_1)}(z) \equiv \psi^{(\nu_1)}(z) - P^{(\nu_1)}L_0 P^{(\nu_1)}$ defined in Eq. (74), i.e.,

$$\delta\widetilde{\psi}^{(\nu_1)}(t) = \frac{1}{2\pi i}\int_C dz\, e^{-izt}\delta\psi^{(\nu_1)}(z), \qquad (2)$$

where the contour C is in the upper-half plane of z for $t>0$ parallel to the real axis and goes from $+\infty$ to $-\infty$. In spite of apparent simplicity of the memory effects associated with the non-Markovian equation (1), the analysis of these effects is a complicated problem and still a controversial issue (see, for example, [19–21]).

To consistently describe quantum decoherence, we need the detailed knowledge of the memory effects. In the complex spectral representations, memory effects associated with non-Markovian equations for reduced systems are represented by a superposition of Markovian equations in each subdynamics. The subdynamics approach then offers a systematic estimation of memory effects.

As a typical setting in studies of quantum decoherence, one usually considers a charged quantum particle (a subsystem) embedded in a field that is in thermodynamic equilibrium. For this case the path-integral method is a useful tool to describe the evolution of the subsystem [22–24]. However, since the system is already in a mixed state as a whole, the so-called collapse of wave functions of the subsystem is trivial for this case (see also [25]). This is simply a "contamination" of the subsystem by the mixed nature of the surrounding field.

In contrast to the path-integral method, our complex spectral representation of the Liouvillian is applicable to more general situations, including those far from equilibrium. For example, we can apply our representation to a pure state satisfying the condition of the *thermodynamic limit*, which ensures the existence of *extensive variables*, such as the total energy being proportional to the size of the system. Indeed, because of this limit density matrices (be they in a pure state or in a mixture) do not belong to the Hilbert space. In the thermodynamic limit we can decompose the time evolution of the initial state into independent Markov processes with finite diffusion rates (subdynamics mentioned above). After the *quantum Zeno period*, only one Markov process dominates, and the initial pure state evolves dynamically into a mixed state. Our theory, therefore, presents a striking nontrivial example of a dynamical collapse of wave functions.

As already mentioned, outside Hilbert space the Liouvillian may acquire complex eigenvalues. Going beyond Hilbert space is a necessary, but not sufficient condition. An additional requirement is the existence of resonances, associated with unstable dynamics.

In the following section we introduce our model that consists of a harmonic oscillator coupled with a field. There we also specify the concept of the thermodynamic limit. We show that due to extensivity of the expectation value of the total Hamiltonian $\langle H \rangle$, the Hilbert norm of states diverges in the thermodynamic limit. We also demonstrate that invariants of motion generated by the Bogoliubov transformation are destroyed in the thermodynamic limit. This prompts us to turn to the Liouvillian formalism that allows us to study the evolution of the density matrix outside Hilbert space.

In Sec. III, we overview the Liouvillian formulation of quantum mechanics. We introduce the projection operators associated to correlations between the harmonic oscillator and the field, and solve the eigenvalue problem of the Liouvillian outside Hilbert space. We show that the eigenvalue problem of the Liouvillian is reduced to the eigenvalue of the collision operator. We also show that the complete set of eigenstates leads to subdynamics.

In Sec. IV, we study the evolution of the density matrix over a long time scale of the relaxation time $t \sim 1/\lambda^2$, where λ is the coupling constant. We derive a Markov kinetic equation for a weak coupling case. In this time scale, contributions from the memory effects are negligible. We present kinetic equations for the reduced density matrices of the harmonic oscillator and the field, respectively.

In Sec. V, we study the evolution of the density matrix over a short time scale $t \ll 1/\lambda^2$. In this time scale one cannot neglect memory effects. We show that the memory effects die out following a power law decay, and the non-Markovian regime of the evolution undergoes a transition into the Markovian regime. The time scale of this transition is known as quantum Zeno time [26]. We show that Zeno time serves as a lower bound for decoherence time.

In the last section we discuss physical implications of our results, and comment on other approaches to the decoherence problem.

II. THE SYSTEM

We consider a system that consists of a one-dimensional quantum harmonic oscillator linearly coupled to a bosonic scalar field $\psi(x)$ through a bilinear interaction. This is a simplified version of a multilevel atom interacting with radiation in the dipole approximation of the multipolar scheme [27]. Extension to arbitrary dimension is straightforward. We shall call the harmonic oscillator "particle" and the quanta of the scalar field "photons" as a convention. The particle with mass M_1 is located at the origin of the space. In the second quantized form, the Hamiltonian is given by (we use atomic units, $\hbar = c = 1$)

$$H = H_0 + \lambda V = \omega_1 a_1^\dagger a_1 + \sum_j \omega_{k_j} a_{k_j}^\dagger a_{k_j} + \lambda \sum_j V_{k_j}(a_1^\dagger + a_1)$$

$$\times (a_{k_j}^\dagger + a_{k_j}), \qquad (3)$$

where $\omega_1 > 0$ and $\omega_{k_j} = |k_j|$. Here λ is a dimensionless coupling constant. We put our system in a one-dimensional box with size L, and impose the usual periodic boundary condition. Then, the spectrum of the field is discrete

$$k_j = j/\Omega,$$

where $\Omega \equiv L/2\pi$ is a volume factor and j is an arbitrary integer. To avoid heavy notations we hereafter abbreviate the index j in the wave vector k_j, and denote the summation

over the wave vector as Σ_k instead of Σ_j. We assume that the volume dependence of V_k is given as $\Omega^{-1/2}$,

$$V_k = \frac{v_k}{\sqrt{\Omega}}, \qquad (4)$$

where v_k is independent of Ω in the limit $\Omega \to \infty$. In the Hamiltonian (3), the usual Dirac creation and annihilation operators for the particle $a_1^\dagger$, a_1 and the field $a_k^\dagger$, a_k obey the canonical commutation relations

$$[a_1, a_1^\dagger] = 1, \quad [a_1, a_k^\dagger] = 0, \quad [a_k, a_{k'}^\dagger] = \delta_{k,k'}, \qquad (5)$$

where $\delta_{k,k'}$ is the Kronecker delta. In the limit $\Omega \to \infty$ we have

$$\sum_k \to \Omega \int dk, \quad \Omega \delta_{k,0} \to \delta(k), \qquad (6)$$

where $\delta(k)$ is the Dirac delta function. The coordinate and momentum operators for the particle are given by

$$\hat{q}_1 = \frac{1}{\sqrt{2M_1\omega_1}}(a_1 + a_1^\dagger), \qquad (7)$$

$$\hat{p}_1 = -i\sqrt{\frac{\omega_1 M_1}{2}}(a_1 - a_1^\dagger). \qquad (8)$$

By introducing the field operator $\phi(x)$ and momentum density operator $\pi(x)$,

$$\phi(x) = \sum_k \left(\frac{1}{2L\omega_k}\right)^{1/2}(a_k^\dagger e^{ikx} + a_k e^{-ikx}), \qquad (9)$$

$$\pi(x) = -i\sum_k \left(\frac{\omega_k}{2L}\right)^{1/2}(a_k^\dagger e^{ikx} - a_k e^{-ikx}), \qquad (10)$$

we may write the Hamiltonian (3) as

$$H = \epsilon_0 + \frac{\hat{p}_1^2}{2M_1} + \frac{M_1\omega_1^2}{2}\hat{q}_1^2 + \frac{1}{2}\int dx\left\{\pi^2(x) + \left(\frac{\partial\phi(x)}{\partial x}\right)^2\right\}$$

$$+ \lambda \int dx\,\hat{q}_1\phi(x)g(x), \qquad (11)$$

where the constant ϵ_0 is the vacuum energy of the particle and the field. Here we have introduced the form factor $g(x)$ to avoid the ultraviolet divergence. The form factor is related to v_k by

$$g(x) = \frac{1}{\Omega}\sum_k \sqrt{\frac{2\omega_k}{\pi M_1\omega_1}}v_k e^{ikx}. \qquad (12)$$

We choose v_k such that the interaction satisfies the following conditions in the limit $\Omega \to \infty$:

$$\int_{-\infty}^{\infty} dk|v_k|^2 < \infty. \qquad (13)$$

Due to the coupling to the field, the particle oscillates with the new frequency $\widetilde{\omega}_1 \equiv \omega_1 + \delta\omega_1$ with $\delta\omega_1 < 0$. We assume that the coupling is so weak, $\lambda \ll 1$, that the frequency shift is much smaller than the unperturbed frequency, i.e., $|\delta\omega_1| \ll \omega_1$.

The complete orthonormal basis of the unperturbed Hamiltonian H_0 is given by

$$H_0|n_1,\{n_F\}\rangle = \left(\omega_1 n_1 + \sum_k \omega_k n_k\right)|n_1,\{n_F\}\rangle, \qquad (14)$$

where $\{n_F\} \equiv \{n_{k_1}, n_{k_2}, \ldots, n_{k_j}\}$ and n_α is the occupation number for the state of particle ($\alpha = 1$) and field ($\alpha = k$), respectively. This basis spans the Hilbert space (Fock space) with the usual Hilbert norm $\||n_1,\{n_F\}\rangle\|^2 = 1$, where $\|\Psi\|^2 \equiv \langle\Psi|\Psi\rangle$.

We describe the evolution of the system in terms of the density matrix in the Liouville space. As we shall see later in Eq. (23), this space is spanned by the basis formed by the dyadic operator generated from the basis in wave function space. The evolution of density matrices is governed by the Liouville–von Neumann equation

$$i\frac{\partial}{\partial t}\rho(t) = L_H\rho(t), \qquad (15)$$

where the *Liouvillian* L_H is the commutator with the Hamiltonian, i.e., $L_H\rho = H\rho - \rho H$. Corresponding to the decomposition of the total Hamiltonian (3) into free and interaction parts, we can decompose the Liouvillian L_H into an unperturbed part L_0 and an interaction L_V as

$$L_H = L_0 + \lambda L_V, \qquad (16)$$

where L_0 and L_V are given by

$$L_0 = H_0 \times 1 - 1 \times H_0 \qquad (17)$$

and

$$L_V = V \times 1 - 1 \times V. \qquad (18)$$

Here, a factorizable *superoperator* $A \times B$ operating on the density matrix, is defined by

$$(A \times B)\rho = A\rho B, \qquad (19)$$

where A and B are linear operators acting on wave functions.

The Liouville space is spanned by linear operators $A, B, \ldots$ in the ordinary wave function space [6]. As usual, the inner product of the Liouville space is defined by

$$\langle\langle A|B\rangle\rangle = \mathrm{Tr}(A^\dagger B), \qquad (20)$$

where $A^\dagger$ is the Hermitian conjugate of the linear operator A in the wave function space. To distinguish a state in the Liouville space from a state in the ordinary wave function space, we shall use a double-ket notation $|A\rangle\rangle$ for the former, and a single-ket notation $|\psi\rangle$ for the latter.

The Hilbert norm $\|\rho\|$ of a state in the Liouville space is defined by

$$\|\rho\|^2 \equiv \langle\langle\rho|\rho\rangle\rangle. \qquad (21)$$

One can introduce the Hermitian conjuate $\mathcal{A}^{\dagger}$ of a given superoperator $\mathcal{A}$ in the Liouville space. One can define Hermitian superoperators and unitary superoperators as usual [6]. The Liouvillian is an example of the Hermitian superoperators, i.e., $L_H^{\dagger} = L_H$.

For the case where the wave function space is spanned by a complete orthonormal basis,

$$\sum_{\alpha} |\alpha\rangle\langle\alpha| = 1, \quad \langle\alpha|\beta\rangle = \delta_{\alpha,\beta}, \qquad (22)$$

the Liouville space is spanned by a complete orthonormal basis of the dyads $|\alpha;\beta\rangle\rangle \equiv |\alpha\rangle\langle\beta|$, i.e.,

$$\sum_{\alpha,\beta} |\alpha;\beta\rangle\rangle\langle\langle\alpha;\beta| = 1, \quad \langle\langle\alpha;\beta|\alpha';\beta'\rangle\rangle = \delta_{\alpha,\alpha'}\delta_{\beta',\beta}. \qquad (23)$$

The matrix element of the usual operator A in the wave function space is given by

$$\langle\langle\alpha;\beta|A\rangle\rangle = \langle\alpha|A|\beta\rangle. \qquad (24)$$

In the occupation number representation the basis for the Liouville space is given by

$$|\alpha;\beta\rangle\rangle = |n_1,\{n_F\};n_1',\{n_F'\}\rangle\rangle. \qquad (25)$$

Let us now specify the meaning of the thermodynamic limit. As an example, let us consider the case where the initial condition for the density matrix $\rho(0)$ is diagonal in the number representation for the field component. This class of initial conditions is often used in quantum Brownian motion [8,22,23]. Then the expectation value of the total energy is given in the continuous spectrum limit $\Omega\to\infty$ by

$$\langle H\rangle = \mathrm{Tr}(H\rho) = \omega_1\langle n_1\rangle + \Omega\int dk\,\omega_k\langle n_k\rangle, \qquad (26)$$

where $\langle A\rangle \equiv \mathrm{Tr}(A\rho)$ denotes the ensemble average of an observable A. Let us assume that $\langle n_k\rangle$ decays rapidly enough for $|k|\to\infty$ to ensure that the integration

$$\int dk\,\omega_k\langle n_k\rangle$$

in Eq. (3) exists. Moreover, assuming a smooth dependence on k for $\langle n_k\rangle$ in the continuous spectrum limit, we may consider two different situations,

$$\langle n_k\rangle \sim O(\Omega^{-1}), \qquad (27a)$$

$$\langle n_k\rangle \sim O(\Omega^0). \qquad (27b)$$

For the case (27a), we have $\langle H\rangle < \infty$ in the limit $\Omega\to\infty$. This is a typical situation considered in the usual scattering theory. On the other hand, for the case (27b), $\langle H\rangle$ is proportional to the volume in the limit $\Omega\to\infty$, i.e., the total energy of the

system is an "extensive" variable. This is the situation corresponding to the thermodynamic limit.

Due to the extensivity of $\langle H\rangle$, a state defined as $|\Psi\rangle \equiv H|n_1,\{n_F\}\rangle$ does not belong to the Hilbert space in the thermodynamic limit, despite the fact that $|n_1,\{n_F\}\rangle$ is an element of the Hilbert space. Indeed, we have for the Hilbert norm in the limit $\Omega\to\infty$,

$$\|\Psi\|^2 = \langle n_1,\{n_F\}|H^2|n_1,\{n_F\}\rangle = \omega_1^2\langle n_1\rangle^2$$
$$+ 2\omega_1\langle n_1\rangle\Omega\int dk\,\omega_k\langle n_k\rangle + \Omega\int dk\,\omega_k^2\langle n_k\rangle$$
$$+ \lambda^2\int dk|v_k|^2(2\langle n_1\rangle+1)(2\langle n_k\rangle+1), \qquad (28)$$

where $\langle n_1\rangle = n_1$ and $\langle n_k\rangle = n_k$. In the nonthermodynamic case (27a), Eq. (28) gives a finite norm, whereas for the thermodynamic limit (27b) this norm diverges. This implies that we need a special consideration to analyze the evolution of our system in the thermodynamic limit.

In the thermodynamic limit, the tagged harmonic mode ω_1 moves according to the field. This corresponds to quantum Brownian motion of the harmonic oscillator.

Recall that our Hamiltonian is given by a bilinear form in annihilation and creation operators. Therefore, one could formally diagonalize the Hamiltonian (3) by introducing dressed annihilation and creation operators B_k and $B_k^{\dagger}$ through a Bogoliubov transformation (see Appendix D)

$$H = \Omega\int dk\,\omega_k B_k^{\dagger}B_k. \qquad (29)$$

Then, we have an infinite set of invariants of motion $\langle B_k^{\dagger}B_k\rangle$ for any given initial condition. For this case, there should be no place for the diffusion-type dissipative processes. However, the formal diagonalization fails in the thermodynamic limit. Indeed, in the continuous spectrum limit the dressed operator B_k involves denominators of the form $(\omega_k - \omega_{k'} \pm i0)^{-1}$, which are distributions (see Appendix D). As a result, the invariants $\langle B_k^{\dagger}B_k\rangle$ involves a product of the distribution $|\omega_k - \omega_{k'} + i0|^{-2}$. In the nonthermodynamic situation, a contribution from a product $\langle B_k^{\dagger}B_k\rangle$ vanishes in the limit $\Omega\to\infty$ because of an extra volume factor Ω^{-1} in Eq. (27a). However, in the thermodynamic situation (27b) this product leads to divergency due to the resonant contribution at $\omega_k = \omega_{k'}$, and the invariants are destroyed.

The appearance of irreversibility due to destruction of invariants of motion in the thermodynamic limit is an important subject, and we shall devote a separate paper to a more detailed discussion [28].

III. COMPLEX SPECTRAL REPRESENTATIONS OF L_H

Since the states representing thermodynamic systems do not belong to Hilbert space, we have to deal with a larger class of density matrices, which have a specific volume dependence in each matrix element, to ensure the existence of extensive and intensive variables in the thermodynamic limit. This family includes canonical equilibrium. In the pre-

vious paper [6] one of us (T.P.) and Prigogine have extended the total Liouvillian L_H to this family of density matrices, and have shown that the Liouvillian may have complex eigenvalues that break time symmetry. The complex spectral representations of the Liouvillian then leads to the ''dynamics of correlations'' describing the non-Markovian evolution in terms of a set of infinite number of Markovian equations. This set includes the Pauli master equation. The detailed derivation of the solution of the eigenvalue problem of the extended Liouvillian as well as the derivation of the kinetic equations has been given in Ref. [6]. For a self-contained presentation, we briefly summarize complex spectral representations. For details the reader should consult [6].

Let us first introduce projection operators that specify correlation components, such as a one-particle distribution of the harmonic oscillator $P_{N_1}^{(\nu_1)}$, correlations between the particle and a single mode k of photons $P_{N_1,N_k}^{(\nu_1,\nu_k)}$, correlations among the particle and two modes k and l of photons $P_{N_1,N_k,N_l}^{(\nu_1,\nu_k,\nu_l)}$, and so on,

$$P_{N_1}^{(\nu_1)} \equiv \sum_{\{n_F\}} |\nu_1^+,\{n_F\};\nu_1^-,\{n_F\}\rangle\rangle\langle\langle \nu_1^+,\{n_F\};\nu_1^-,\{n_F\}|,$$
(30)

$$P_{N_1,N_k}^{(\nu_1,\nu_k)} \equiv \sum_{\{n_F\}(k)} |\nu_1^\dagger,\nu_k^\dagger,\{n_F\}_{(k)};\nu_1^-,\nu_k^-,\{n_F\}_{(k)}\rangle\rangle$$
$$\times \langle\langle \nu_1^\dagger,\nu_k^\dagger,\{n_F\}_{(k)};\nu_1^-,\nu_k^-,\{n_F\}_{(k)}|(1-\delta_{\nu_k,0}),$$
(31)

$$P_{N_1,N_k,N_l}^{(\nu_1,\nu_k,\nu_l)} \equiv \sum_{\{n_F\}(k,l)} |\nu_1^\dagger,\nu_k^\dagger,\nu_l^\dagger,\{n_F\}_{(k,l)};\nu_1^-,\nu_k^-,\nu_l^-$$
$$\times\{n_F\}_{(k,l)}\rangle\rangle\langle\langle \nu_1^\dagger,\nu_k^\dagger,\nu_l^\dagger,\{n_F\}_{(k,l)};\nu_1^-,\nu_k^-,\nu_l^\dagger$$
$$\times\{n_F\}_{(k,l)}|(1-\delta_{\nu_k,0})(1-\delta_{\nu_l,0}).$$
(32)

Since $P_{N_1,N_k,N_l}^{(\nu_1,\nu_k,\nu_l)}$ involves more modes of the particle and the field, this subspace is *more correlated* than the subspace $P_{N_1,N_k}^{(\nu_1,\nu_k)}$. Here $\{n_F\}_{(k,\ldots,l)}$ means that the components of the field $n_k,\ldots,n_l$ are excluded from the set $\{n_F\} = \{n_{k_1},n_{k_2},n_{k_3},\ldots\}$. Moreover,

$$\nu_j^+ \equiv N_j + \frac{\nu_j}{2} = n_j,$$
(33)

$$\nu_j^- \equiv N_j - \frac{\nu_j}{2} = m_j,$$
(34)

for integers $2N_j \equiv (n_j+m_j)$ and integers $\nu_j \equiv n_j-m_j$, where $j=1$ or k.

We introduce operators

$$P^{(\nu_1)} \equiv \sum_{N_1} P_{N_1}^{(\nu_1)}, \quad P^{(\nu_1\nu_k)} \equiv \sum_{N_1}\sum_{N_k} P_{N_1,N_k}^{(\nu_1,\nu_k)}, \quad \ldots.$$
(35)

These are eigenprojectors of L_0,

$$L_0 P^{(\nu_1\nu_k\cdots\nu_l)} = P^{(\nu_1\nu_k\cdots\nu_l)} L_0 = (\nu_1\omega_1 + \nu_k\omega_k + \cdots$$
$$+ \nu_l\omega_l)P^{(\nu_1\nu_k\cdots\nu_l)}$$
(36)

and satisfy the completeness and orthonormality conditions

$$\sum_{\nu_1} P^{(\nu_1)} + \sum_k\sum_{\nu_1,\nu_k} P^{(\nu_1\nu_k)} + \sum_{k,l}\sum_{\nu_1,\nu_k,\nu_l} P^{(\nu_1\nu_k\nu_l)} + \cdots = 1,$$
(37)

$$P^{(\nu_1\nu_k\cdots\nu_l)}P^{(\mu_1\mu_k\cdots\mu_l)} = P^{(\nu_1\nu_k\cdots\nu_l)}\delta_{\nu_1,\mu_1}\delta_{\nu_k,\mu_k}\cdots\delta_{\nu_l,\mu_l}.$$
(38)

The eigenprojectors (35) are Hermitian operators in Liouville space

$$(P^{(\nu_1\nu_k\cdots\nu_l)})^\dagger = P^{(\nu_1\nu_k\cdots\nu_l)}.$$
(39)

Diagonal components of density matrices are associated to the ''vacuum of correlation'' subspace and are singled out by $P^{(0)}$ (i.e., $P^{(\nu_1)}$ with $\nu_1=0$).

To avoid heavy notation, we introduce the superscript $\nu \equiv \{\nu_1\nu_k\ldots\nu_l\}$, and express Eq. (36) as

$$L_0 P^{(\nu)} = P^{(\nu)}L_0 = w^{(\nu)}P^{(\nu)},$$
(40)

where $w^{(\nu)} \equiv \nu_1\omega_1 + \nu_k\omega_k + \cdots + \nu_l\omega_l$. We also abbreviate summation over the wave vector of photons in completeness and orthogonality relation, i.e., we write Eqs. (37) and (38) as

$$\sum_\nu P^{(\nu)} = 1, \quad P^{(\nu)}P^{(\mu)} = \delta_{\mu,\nu}P^{(\nu)}.$$
(41)

We also introduce the complements

$$Q^{(\nu)} \equiv 1 - P^{(\nu)}.$$
(42)

We have

$$(P^{(\nu)})^2 = P^{(\nu)}, \quad (Q^{(\nu)})^2 = Q^{(\nu)}, \quad P^{(\nu)}Q^{(\nu)} = Q^{(\nu)}P^{(\nu)} = 0$$
(43)

and

$$L_0 P^{(\nu)} = P^{(\nu)}L_0, \quad L_0 Q^{(\nu)} = Q^{(\nu)}L_0.$$
(44)

Let us now consider the eigenvalue problem of the Liouvillian[1] [6],

---

[1] We formulate the eigenvalue problem of L_H in the thermodynamic limit. In this limit, special care is necessary for a general Hamiltonian, as the perturbed Liouvillian L_V usually gives rise to divergence due to disconnected processes (see connected and disconnected diagrams in Refs. [6,17]). However, in the case of our Hamiltonian we do not encounter this divergence as all processes are connected to the tagged mode ω_1 through the specific form of interaction involving a_1 or $a_1^\dagger$.

$$L_H|F_\alpha^{(\nu)}\rangle\rangle = z_\alpha^{(\nu)}|F_\alpha^{(\nu)}\rangle\rangle, \quad \langle\langle\widetilde{F}_\alpha^{(\nu)}|L_H = \langle\langle\widetilde{F}_\alpha^{(\nu)}|z_\alpha^{(\nu)}. \tag{45}$$

The index α together with ν is a parameter characterizing the eigenfunctions. For the unstable case some of the eigenvalues $z_\alpha^{(\nu)}$ may be complex. As mentioned before, this is only possible if we consider the eigenvalue problem that corresponds to an extension of L_H outside Hilbert space. As a result, the left eigenstates are generally not the same as the Hermitian conjugate of the right eigenstates.

Because the eigenvalues $z_\alpha^{(\nu)}$ are complex, the time evolution of the system splits into two semigroups. For the semigroup corresponding to $t>0$, the eigenstates are associated with the eigenvalues with $\mathrm{Im}\,z_\alpha^{(\nu)} \leq 0$ (including the case $\mathrm{Im}\,z_\alpha^{(\nu)} < 0$) and equilibrium is reached in our future for $t \rightarrow +\infty$, while for the other the eigenvalues are the complex conjugate of $z_\alpha^{(\nu)}$ and equilibrium is reached in our past. Experience shows that all irreversible processes have the same time orientation. To be self-consistent, we have to choose the semigroup oriented towards our future.

The right and left eigenstates satisfy biorthogonality and bicompleteness relations,

$$\langle\langle\widetilde{F}_\alpha^{(\nu)}|F_\beta^{(\mu)}\rangle\rangle = \delta_{\nu,\mu}\delta_{\alpha,\beta}, \quad \sum_\nu\sum_\alpha|F_\alpha^{(\nu)}\rangle\rangle\langle\langle\widetilde{F}_\alpha^{(\nu)}| = 1. \tag{46}$$

We consider the case where the eigenstates are analytic with respect to the coupling constant λ, i.e.,

$$\lim_{\lambda\to 0}|F_\alpha^{(\nu)}\rangle\rangle = \lim_{\lambda\to 0}P^{(\nu)}|F_\alpha^{(\nu)}\rangle\rangle, \quad \lim_{\lambda\to 0}\langle\langle\widetilde{F}_\alpha^{(\nu)}| = \lim_{\lambda\to 0}\langle\langle\widetilde{F}_\alpha^{(\nu)}|P^{(\nu)} \tag{47}$$

and

$$\lim_{\lambda\to 0}z_\alpha^{(\nu)} = w^{(\nu)}, \tag{48}$$

where $w^{(\nu)}$ is defined in Eq. (40).

Using $P^{(\nu)} + Q^{(\nu)} = 1$ in Eq. (45) one can find the $Q^{(\nu)}$ component of the eigenstates as

$$Q^{(\nu)}|F_\alpha^{(\nu)}\rangle\rangle = \mathcal{C}^{(\nu)}(z_\alpha^{(\nu)})P^{(\nu)}|F_\alpha^{(\nu)}\rangle\rangle, \tag{49}$$

where the "creation-of-correlation operator" (creation operator in short) is defined as

$$\mathcal{C}^{(\nu)}(z) \equiv \frac{-1}{Q^{(\nu)}L_HQ^{(\nu)} - z}Q^{(\nu)}\lambda L_V P^{(\nu)}. \tag{50}$$

This operator is an "off-diagonal operator" as it describes an "off-diagonal transition" between the different subspace $Q^{(\nu)}$ and $P^{(\nu)}$. Care has to be taken in the analytic continuation of z to have the time evolution approaching equilibrium in our future $t>0$.

Substituting Eq. (49) into the eigenvalue equation (45), we obtain

$$\psi^{(\nu)}(z_\alpha^{(\nu)})|u_\alpha^{(\nu)}\rangle\rangle = z_\alpha^{(\nu)}|u_\alpha^{(\nu)}\rangle\rangle, \tag{51}$$

where

$$|u_\alpha^{(\nu)}\rangle\rangle \equiv [N_\alpha^{(\nu)}]^{-1/2}P^{(\nu)}|F_\alpha^{(\nu)}\rangle\rangle. \tag{52}$$

Here $N_\alpha^{(\nu)}$ is a normalization constant and $\psi^{(\nu)}(z)$ are the *collision operators* defined by

$$\psi^{(\nu)}(z) \equiv P^{(\nu)}L_0P^{(\nu)} + P^{(\nu)}\lambda L_V\mathcal{C}^{(\nu)}(z)P^{(\nu)}. \tag{53}$$

The collision operator is a "diagonal operator" since it describes a "diagonal transition" within the same subspace $P^{(\nu)}$. As indicated in Eq. (51), the eigenvalue problem of the Liouville operator is then reduced to the eigenvalue problem of the collision operators that have the same eigenvalues $z^{(\nu)}$ as L_H. The eigenvalue equation (51) is "nonlinear" as the eigenvalue appears in the collision operator.

Using Eq. (49), we obtain the right eigenstates of L_H from the corresponding right eigenstates of $\psi^{(\nu)}(z_\alpha^{(\nu)})$ as

$$|F_\alpha^{(\nu)}\rangle\rangle = [N_\alpha^{(\nu)}]^{1/2}[P^{(\nu)} + \mathcal{C}^{(\nu)}(z_\alpha^{(\nu)})]|u_\alpha^{(\nu)}\rangle\rangle. \tag{54}$$

A construction parallel to the above, leads to the left eigenstates of L_H with the same eigenvalues $z_\alpha^{(\nu)}$,

$$\langle\langle\widetilde{F}_\alpha^{(\nu)}| = \langle\langle\widetilde{v}_\alpha^{(\nu)}|[P^{(\nu)} + \mathcal{D}^{(\nu)}(z_\alpha^{(\nu)})][N_\alpha^{(\nu)}]^{1/2}, \tag{55}$$

where the "destruction operator" is defined by

$$\mathcal{D}^{(\nu)}(z) \equiv P^{(\nu)}\lambda L_V Q^{(\nu)}\frac{1}{z - Q^{(\nu)}L_HQ^{(\nu)}}, \tag{56}$$

and $\langle\langle\widetilde{v}_\alpha^{(\nu)}| \equiv \langle\langle\widetilde{F}_\alpha^{(\nu)}|P^{(\nu)}N_\alpha^{(\nu)-1/2}$ are the left eigenstates of the collision operator,

$$\langle\langle\widetilde{v}_\alpha^{(\nu)}|\psi^{(\nu)}(z_\alpha^{(\nu)}) = \langle\langle\widetilde{v}_\alpha^{(\nu)}|z_\alpha^{(\nu)}. \tag{57}$$

Since the collision operator depends on the eigenvalue $z_\alpha^{(\nu)}$, the state $\langle\langle\widetilde{v}_\alpha^{(\nu)}|$ is generally not biorthogonal to $|u_\alpha^{(\nu)}\rangle\rangle$. Assuming, however, bicompleteness of these states in each $P^{(\nu)}$ subspace, we may always construct sets of states $\{\langle\langle\widetilde{u}_\alpha^{(\nu)}|\}$ and $\{|v_\alpha^{(\nu)}\rangle\rangle\}$ biorthogonal to $\{|u_\alpha^{(\nu)}\rangle\rangle\}$ and $\{\langle\langle\widetilde{v}_\alpha^{(\nu)}|\}$, respectively,

$$\langle\langle\widetilde{u}_\alpha^{(\nu)}|u_\beta^{(\mu)}\rangle\rangle = \delta_{\nu,\mu}\delta_{\alpha,\beta}, \quad \sum_\alpha|u_\alpha^{(\nu)}\rangle\rangle\langle\langle\widetilde{u}_\alpha^{(\nu)}| = P^{(\nu)} \tag{58}$$

and similarly for $|v_\alpha^{(\nu)}\rangle\rangle$ and $\langle\langle\widetilde{v}_\alpha^{(\nu)}|$.

In order to connect the kinetic theory to the eigenvalue problem of L_H, we introduce the "global" creation and destruction operators

$$\mathbf{C}^{(\nu)} \equiv \sum_\alpha \mathcal{C}^{(\nu)}(z_\alpha^{(\nu)})|u_\alpha^{(\nu)}\rangle\rangle\langle\langle\widetilde{u}_\alpha^{(\nu)}|, \tag{59}$$

$$\mathbf{D}^{(\nu)} \equiv \sum_\alpha |v_\alpha^{(\nu)}\rangle\rangle\langle\langle\widetilde{v}_\alpha^{(\nu)}|\mathcal{D}^{(\nu)}(z_\alpha^{(\nu)}). \tag{60}$$

Then, we can write the eigenstates as

$$|F_\alpha^{(\nu)}\rangle\rangle = [N_\alpha^{(\nu)}]^{1/2}(P^{(\nu)}+\mathbf{C}^{(\nu)})|u_\alpha^{(\nu)}\rangle\rangle, \qquad (61)$$

$$\langle\langle \tilde{F}_\alpha^{(\nu)}| = \langle\langle \tilde{v}_\alpha^{(\nu)}|(P^{(\nu)}+\mathbf{D}^{(\nu)})[N_\alpha^{(\nu)}]^{1/2}. \qquad (62)$$

The normalization constant may be found from the biorthonormality condition of the eigenstates,

$$[N_\alpha^{(\nu)}]^{-1} = \langle\langle \tilde{v}_\alpha^{(\nu)}|[A^{(\nu)}]^{-1}|u_\alpha^{(\nu)}\rangle\rangle, \qquad (63)$$

where

$$A^{(\nu)} = [(P^{(\nu)}+\mathbf{D}^{(\nu)})(P^{(\nu)}+\mathbf{C}^{(\nu)})]^{-1} = [(P^{(\nu)}$$
$$+\mathbf{D}^{(\nu)}\mathbf{C}^{(\nu)})]^{-1}. \qquad (64)$$

We have

$$A^{(\nu)} = \sum_\alpha |u_\alpha^{(\nu)}\rangle\rangle N_\alpha^{(\nu)}\langle\langle \tilde{v}_\alpha^{(\nu)}|. \qquad (65)$$

The global collision operators associated with the creation operator $\mathbf{C}^{(\nu)}$ are given by

$$\theta^{(\nu)} \equiv \sum_\alpha \psi^{(\nu)}(z_\alpha^{(\nu)})|u_\alpha^{(\nu)}\rangle\rangle\langle\langle \tilde{u}_\alpha^{(\nu)}|. \qquad (66)$$

Then we have

$$\theta^{(\nu)}|u_\alpha^{(\nu)}\rangle\rangle = z_\alpha^{(\nu)}|u_\alpha^{(\nu)}\rangle\rangle. \qquad (67)$$

Substituting Eq. (53) into Eq. (56), we have

$$\theta^{(\nu)} = P^{(\nu)}L_0 P^{(\nu)} + P^{(\nu)}\lambda L_V \mathbf{C}^{(\nu)} P^{(\nu)}. \qquad (68)$$

Now we define the dressed projection operators in each subspace in terms of eigenstates (61) and (62),

$$\Pi^{(\nu)} \equiv \sum_\alpha |F_\alpha^{(\nu)}\rangle\rangle\langle\langle \tilde{F}_\alpha^{(\nu)}|. \qquad (69)$$

This leads to the relation

$$e^{-iL_H t}\Pi^{(\nu)} = \Pi^{(\nu)}e^{-iL_H t} = (P^{(\nu)}+\mathbf{C}^{(\nu)})e^{-i\theta^{(\nu)}t}A^{(\nu)}$$
$$\times (P^{(\nu)}+\mathbf{D}^{(\nu)}). \qquad (70)$$

From Eqs. (45) and (46), we have

$$L_H\Pi^{(\nu)} = \Pi^{(\nu)}L_H, \quad \sum_\nu \Pi^{(\nu)} = 1, \quad \Pi^{(\nu)}\Pi^{(\mu)} = \Pi^{(\nu)}\delta_{\nu,\mu}. \qquad (71)$$

Hence, $\Pi^{(\nu)}$ is a generalization of $P^{(\nu)}$ for the total Liouvillian L_H. However, Eq. (69) shows that $\Pi^{(\nu)}$ is not a Hermitian superoperator, i.e.,

$$(\Pi^{(\nu)})^\dagger \neq \Pi^{(\nu)}. \qquad (72)$$

Equations (70) and (71) show that each correlation subspace $\Pi^{(\nu)}$ evolves independently of other subspaces. For this reason $\Pi^{(\nu)}$ is associated with *subdynamics*. We call the component $P^{(\nu)}\rho^{(\nu)}$ the "privileged" component of $\rho^{(\nu)}$

$\equiv \Pi^{(\nu)}\rho$. Taking the time derivative of the privileged component in Eq. (70), we obtain the "Markovian" kinetic equation

$$i\frac{\partial P^{(\nu)}\rho^{(\nu)}(t)}{\partial t} = \theta^{(\nu)}P^{(\nu)}\rho^{(\nu)}(t). \qquad (73)$$

There is an infinite set of Markovian processes associated with each $\Pi^{(\nu)}$ subspace.

In this work we are interested in the weak coupling limit. For this case, the relation between the original collision operator $\psi^{(\nu)}(z)$ and the global collision operator $\theta^{(\nu)}$ takes a simple form. Indeed, a series expansion of the creation operator (50) in powers of λ starts from λ. Keeping the interaction term up to λ^2 in Eq. (53), we have[2]

$$\psi^{(\nu)}(z_\alpha^{(\nu)}) = P^{(\nu)}L_0 P^{(\nu)} + \lambda^2 \psi_2^{(\nu)}(w^{(\nu)}+i0) + O(\lambda^4). \qquad (74)$$

Here $\psi_2^{(\nu)}(z) = P^{(\nu)}L_V C_1^{(\nu)}(z)P^{(\nu)}$ and $C_1^{(\nu)}(z)$ is the first-order contribution in $C^{(\nu)}(z)$, i.e.,

$$C_1^{(\nu)}(z) = \frac{-1}{L_0-z}Q^{(\nu)}\lambda L_V P^{(\nu)} \qquad (75)$$

and we have analytically continued $z_\alpha^{(\nu)}$ from the upper-half plane to guarantee that the time evolution is oriented to our future $t>0$. Substituting Eq. (74) into Eq. (68), we obtain

$$\theta^{(\nu)} = P^{(\nu)}L_0 P^{(\nu)} + \lambda^2 \psi_2^{(\nu)}(w^{(\nu)}+i0) + O(\lambda^4), \qquad (76)$$

where we have used the second relation in Eq. (58). This result allows us to calculate Eq. (76) in Eq. (73) the lowest-order approximation in λ. We also have the following perturbation expansions:

$$\mathbf{C}^{(\mu)} = \lambda C_1^{(\mu)}(w^{(\mu)}) + O(\lambda^2), \qquad (77)$$

$$\mathbf{D}^{(\mu)} = \lambda D_1^{(\mu)}(w^{(\mu)}) + O(\lambda^2), \qquad (78)$$

$$A^{(\mu)} = P^{(\mu)} - \lambda^2 D_1^{(\mu)}(w^{(\mu)})C_1^{(\mu)}(w^{(\mu)}) + O(\lambda^4), \qquad (79)$$

with a suitable analytic continuation of $w^{(\mu)}$ (see Appendix B). Here the subscript n in each superoperator means the nth order contribution in the λ expansion of a superoperator.

IV. MARKOVIAN KINETIC EQUATION

Let us now apply the formulation developed in the preceding section to describe the evolution of our harmonic oscillator governed by the Hamiltonian (3). We assume that the initial state of ρ is diagonal in the field component in the number representation[3]

---

[2]For the Hamiltonian (3), there appear only terms of even order in λ in the expansion of the collision operator (53). In general, terms of odd power in λ may also appear in the expansion.

[3]$P^{(\nu_1)}$ is defined in Eq. (35). It should not be confused with the abbreviated notation $P^{(\nu)}$ introduced in Eq. (41).

$$\rho(0) = \sum_{\nu_1} P^{(\nu_1)}\rho(0)$$

$$= \sum_{n_1,m_1} \sum_{\{n_F\}} |n_1,\{n_F\}\rangle \rho(n_1,\{n_F\};m_1,\{n_F\},0)$$

$$\times \langle m_1,\{n_F\}|. \tag{80}$$

Moreover, we assume that the initial condition is factorizable into a product of the particle state and the field state as

$$\rho(n_1,\{n_F\};m_1,\{n_F\},0) = f_{n_1,m_1}(0) \prod_k f_k(n_k,n_k,0) \tag{81}$$

with the normalization conditions

$$\sum_{n_1} f_{n_1,n_1}(0) = 1, \tag{82}$$

$$\sum_{n_k} f_k(n_k,n_k,0) = 1, \tag{83}$$

where we have introduced the reduced density matrix for the particle,

$$f_{n_1,m_1}(t) \equiv \langle n_1 | \hat{f}(t) | m_1 \rangle \tag{84}$$

with

$$\hat{f}(t) \equiv \mathrm{Tr}_F[P^{(\nu_1)}\rho(t)], \tag{85}$$

and for the field

$$f_k(n_k,n_k,t) \equiv \langle n_k | \mathrm{Tr}_{p,F_{(k)}}[P^{(0)}\rho(t)] | n_k \rangle, \tag{86}$$

where $P^{(0)}$ means $P^{(\nu_1)}$ with $\nu_1 = 0$, and Tr_F means that the partial trace is taken with respect to all field components, while $\mathrm{Tr}_{p,F_{(k)}}$ means that the partial trace is taken with respect to all components except for the kth-mode of the field.

One of the possible choices of initial conditions is to assume that the field is in unperturbed thermodynamic equilibrium,

$$f_k(n_k,n_k,0) = \frac{e^{-\beta n_k \omega_k}}{Q}, \tag{87}$$

where $Q \equiv [1 - \exp(-\beta\omega_k)]^{-1}$ is the partition function and $\beta = (k_B T)^{-1}$. This case has been treated extensively in the literature [8,9,22–24]. For this choice of initial conditions, a system is in a mixed state near thermal equilibrium. For this situation, the path-integral method is applicable to study the evolution of the system [8,22,23].

Another interesting choice of initial conditions is given by a pure state $\rho(0) = |\psi_1\rangle\langle\Psi_1| \otimes |\{n_F^0\}\rangle\langle\{n_F^0\}|$. Then we have

$$f_{n_1,m_1}(0) = \langle n_1 | \psi_1 \rangle \langle \Psi_1 | m_1 \rangle, \quad f_k(n_k,n_k,0) = \delta_{n_k,n_k^0}, \tag{88}$$

where we have assumed that in the limit $\Omega \rightarrow \infty$

$$n_k^0 \sim O(\Omega^0) \quad \text{and} \quad \int dk\, n_k^0 < \infty, \tag{89}$$

i.e., the thermodynamic condition is satisfied [see Eq. (27b)]. For the choice of initial conditions (88), the system is far from equilibrium. Our complex spectral representation is applicable to both near equilibrium (87) and far from equilibrium (88), whereas the path-integral method is not a useful tool to describe the evolution of the system far from equilibrium. Of special importance is the case (88), since as we will show, even starting from this pure state, the system evolves to a mixed state from the pure state in the thermodynamic limit. Hence, the wave function collapses dynamically in this limit [29].

Let us now evaluate the time evolution of the component $P^{(\nu_1)}\rho(t)$,

$$P^{(\nu_1)}\rho(t) = P^{(\nu_1)}\rho^{(\nu_1)}(t) + P^{(\nu_1)}\rho_{np}^{(\nu_1)}(t), \tag{90}$$

where $P^{(\nu_1)}\rho^{(\nu_1)}$ is the privileged component in $\Pi^{(\nu_1)}$ subspace, whereas $\rho_{np}^{(\nu_1)} \equiv Q^{(\nu_1)}\rho$ is the nonprivileged component [see Eq. (70)],

$$P^{(\nu_1)}\rho^{(\nu_1)}(t) = P^{(\nu_1)}e^{-i\theta^{(\nu_1)}t}A^{(\nu_1)}P^{(\nu_1)}\rho(0), \tag{91}$$

$$P^{(\nu_1)}\rho_{np}(t) = \sum_{\mu(\neq\nu_1)} P^{(\nu_1)}\mathbf{C}^{(\mu)}e^{-i\theta^{(\mu)}t}A^{(\mu)}\mathbf{D}^{(\mu)}P^{(\nu_1)}\rho(0). \tag{92}$$

In the lower-order approximation in λ, we have

$$\theta_0^{(\nu_1)} = \nu_1\omega_1 P^{(\nu_1)}, \tag{93}$$

$$\lambda^2\theta_2^{(\nu_1)} = \lambda^2 P^{(\nu_1)}L_V\mathbf{C}_1^{(\nu_1)}P^{(\nu_1)}, \tag{94}$$

$$\lambda\mathbf{C}_1^{(\nu_1)} = \lambda\frac{-1}{L_0 - \nu_1\omega_1 - i0}Q^{(\nu_1)}L_VP^{(\nu_1)}, \tag{95}$$

$$\lambda\mathbf{D}_1^{(\nu_1)} = \lambda P^{(\nu_1)}L_VQ^{(\nu_1)}\frac{1}{\nu_1\omega_1 - L_0 + i0}. \tag{96}$$

First, let us consider a long time scale $t \sim \lambda^{-2}$, which corresponds to the relaxation time scale. Later, we will consider a short time scale where $t \ll \lambda^{-2}$. In the long time scale we can approximate the privileged component by

$$P^{(\nu_1)}\rho(t) \approx P^{(\nu_1)}\exp[-i(\nu_1\omega_1 + \lambda^2\theta_2^{(\nu_1)})t]P^{(\nu_1)}\rho(0), \tag{97}$$

where $A^{(\nu_1)}$ has been approximated by $P^{(\nu_1)}$ by neglecting higher-order terms in λ, and the collision operator $\theta^{(\nu_1)}$ is approximated by $\nu_1\omega_1 P^{(\nu_1)} + \lambda^2\psi_2^{(\nu_1)}(\nu_1\omega_1 + i0)$. This corresponds to the so-called $\lambda^2 t$ approximation [17].[4]

---

[4]Irreversibility is not introduced by this approximation, but is a result of the nonvanishing imaginary part of the complex eigenvalue $z_\alpha^{(\nu_1)}$. This approximation is a way to evaluate the imaginary

Equation (92) with Eqs. (77) and (78) shows that the dominant contribution to the evolution of the non-privileged component (with $\mu \neq \nu_1$) in each $\Pi^{(\mu)}$ subspace starts with the second-order term in λ. Therefore, we can neglect the nonprivileged components for the time scale of $t \sim \lambda^{-2}$, i.e., we have

$$P^{(\nu_1)}\rho(t) \approx P^{(\nu_1)}\rho^{(\nu_1)}(t).$$ (98)

Taking the time derivative of Eqs. (84) and (86) with Eq. (97), we have for $t \sim \lambda^{-2}$,

$$i\frac{\partial}{\partial t}f_k(n_k,n_k,t) = \lambda^2 \langle n_k| \mathrm{Tr}_{p,F_k}[\theta_2^{(0)}P^{(0)}\rho^{(1)}(t)]|n_k\rangle,$$ (99)

$$i\frac{\partial}{\partial t}f_{n_1,m_1}(t) = (n_1-m_1)\omega_1 f_{n_1,m_1}(t)$$
$$+ \lambda^2 \langle n_1| \mathrm{Tr}_F[\theta_2^{(\nu_1)}P^{(\nu_1)}\rho^{(1)}(t)]|m_1\rangle.$$ (100)

Using Eq. (95) with the expressions (3), (30)–(32), we obtain an explicit expression for the right hand side (RHS) of Eqs. (99) and (100). However, since calculation is lengthy, we have presented the calculation in Appendix A, and here we display only the results. We obtain the Markov equations for $t \sim \lambda^{-2}$ as

$$\frac{\partial}{\partial t}f_k(n_k,n_k,t) = \frac{\lambda^2 \gamma_k}{\Omega}[n_k \langle n_1 \rangle_t f_k(n_k-1,n_k-1;t) - (\langle n_1 \rangle_t$$
$$+ n_k + 2\langle n_1 \rangle_t n_k)f_k(n_k,n_k,t) + (\langle n_1 \rangle_t + 1)$$
$$\times (n_k+1)f_k(n_k+1,n_k+1,t)]$$ (101)

and

$$\frac{\partial}{\partial t}f_{n_1,m_1}(t) - i\tilde{\omega}_1(m_1-n_1)f_{n_1,m_1}(t)$$
$$= \lambda^2 \int dk\, \gamma_k \left\{ \sqrt{n_1 m_1}\langle n_k \rangle_t f_{n_1-1,m_1-1}(t) \right.$$
$$- \left[(n_1+m_1+1)\langle n_k \rangle_t + \frac{1}{2}(n_1+m_1)\right]f_{n_1,m_1}(t)$$
$$\left. + \sqrt{(n_1+1)(m_1+1)}(\langle n_k \rangle_t + 1)f_{n_1+1,m_1+1}(t)\right\}.$$ (102)

Here

_____

part of $z_\alpha^{(\nu_1)}$ up to the second-order contribution in λ; cf. Appendix D where the decay rate of spontaneous emission of an excited state is evaluated in the $\lambda^2 t$ limit (see also Ref. [4]). However, care has to be taken when the unperturbed discrete spectrum is located near the lower bound of the continuous spectrum of the field.

$$\gamma_k \equiv 2\pi |v_k|^2 \delta(\omega_k - \omega_1)$$ (103)

and the renormalized frequency of the particle is

$$\tilde{\omega}_1 \equiv \omega_1 + \lambda^2 \delta\omega_1$$ (104)

with the frequency shift

$$\lambda^2 \delta\omega_1 \equiv 2\pi\lambda^2 \, \mathrm{P}\int dk |v_k|^2 \frac{2\omega_1}{\omega_1^2 - \omega_k^2},$$ (105)

where P stands for the principal part. The quantities $\langle n_1 \rangle_t$ and $\langle n_k \rangle_t$ are the average numbers of quanta of the particle and the field, respectively, at time t. To obtain Eqs. (101) and (102) we have used the factorization property (81) that persists for any $t > 0$ [17].

The RHS of Eqs. (102) and (101) are dissipative terms. These results show that dissipation comes from the contribution at the resonance $\omega_k = \omega_1$ [see Eq. (103)].

The RHS of Eq. (101) is proportional to $1/\Omega$. Hence, if we start with nonvanishing $f_k(n_k,n_k,0)$ of the order Ω^0, then we can neglect the expression in the RHS in the thermodynamic limit as $\Omega \to \infty$. Examples are the finite temperature case in Eq. (87), or a pure state with Eq. (88). For these cases, the photon distribution function is invariant in time, i.e.,

$$f_k(n_k,n_k,t) = f_k(n_k,n_k,0).$$ (106)

However, if we start at zero temperature, i.e., $T = 0$ in Eq. (87) so that there is no photon at $t = 0$ [or $n_k^0 \sim O(\Omega^{-1})$ in Eq. (88)], then one cannot neglect the expression in the RHS of Eq. (101). This corresponds to the case of spontaneous emission of the photon by the excited particle. In this case, $f_k(n_k,n_k,t)$ changes in time and $\langle n_k \rangle_t$ approaches the line shape of the emitted photon as $t \to \infty$. Indeed, our kinetic equations (102) and (101) lead to the Pauli-type equation, which corresponds to the well-known Uhling-Uhlenbeck equations for the average number of particles and the field for the case of nonlinear interaction, i.e.,

$$\frac{\partial}{\partial t}\langle n_1 \rangle_t = \frac{\lambda^2}{\Omega} \sum_k \gamma_k[\langle n_k \rangle_t - \langle n_1 \rangle_t],$$ (107)

$$\frac{\partial}{\partial t}\langle n_k \rangle_t = \frac{\lambda^2}{\Omega} \gamma_k[\langle n_1 \rangle_t - \langle n_k \rangle_t].$$ (108)

If the thermodynamic limit is fulfilled, one can again neglect the RHS of Eq. (108), i.e., $\langle n_k \rangle_t$ is an invariant of motion [see Eq. (111)]. On the other hand, for a nonthermodynamic situation such as $T = 0$, the RHS of Eq. (108) is not negligible in describing a decay of the excited particle. Indeed, for the initial condition $\langle n_k \rangle_0 = 0$, the solution of Eqs. (107) and (108) is given by

$$\langle n_1 \rangle_t = e^{-\lambda^2 \gamma t}\langle n_1 \rangle_0, \quad \langle n_k \rangle_t = \frac{1}{\Omega}\frac{\gamma_k}{\gamma}(1 - e^{-\lambda^2 \gamma t})\langle n_1 \rangle_0,$$ (109)

where

$$\gamma \equiv \frac{1}{\Omega} \sum_k \gamma_k, \tag{110}$$

which leads to the well-known line shape proportional to $\delta(\omega_k - \omega_1)$ in the $\lambda^2 t$ approximation. Note that a non-negligible contribution to the RHS of Eq. (101) for the zero temperature case is essential in order to keep the density matrix in a pure state for arbitrary time $t > 0$ by starting with a pure initial state.

We first restrict ourselves to the thermodynamic case (we will discuss the nonthermodynamic case, i.e., $T = 0$ in Sec. VI). Then, the average number of photons $\langle n_k \rangle_t$ in the kth mode of the field is an invariant of motion, i.e.,

$$\langle n_k \rangle_t = \langle n_k \rangle_0 = \sum_{n_k} n_k f_k(n_k, n_k; 0). \tag{111}$$

Then we obtain a closed kinetic equation (102) for the reduced particle distribution function $f(n_1, m_1, t)$, which is one of the main results in this paper. However, in order to discuss physical consequences of our kinetic equation, we have to justify our approximation (97), where we have neglected contributions from all nonprivileged components, coming from other subdynamics. These components are responsible for memory effects (i.e., non-Markovian processes) that are essential for estimation of the decoherence time scale. In the following section, we will analyze the memory effects in detail.

Before going to the next section, let us present our kinetic equation (102) for the case (111) in other useful representations. First, Eq. (102) leads to the following master equation for the operator $\hat{f}(t)$ defined in Eq. (85) [9,29–32],

$$\frac{\partial}{\partial t} \hat{f}(t) = -i\tilde{\omega}_1 [a_1^\dagger a_1, \hat{f}(t)] + \frac{\lambda^2}{2} \int dk \, \gamma_k (\langle n_k \rangle_0 + 1)$$

$$\times [2a_1 \hat{f}(t) a_1^\dagger - a_1^\dagger a_1 \hat{f}(t) - \hat{f}(t) a_1^\dagger a_1]$$

$$+ \frac{\lambda^2}{2} \int dk \, \gamma_k \langle n_k \rangle_0 [2a_1^\dagger \hat{f}(t) a_1 - a_1 a_1^\dagger \hat{f}(t)$$

$$- \hat{f}(t) a_1 a_1^\dagger]. \tag{112}$$

Second, in the coordinate representation

$$f(q_1, q_1', t) \equiv \sum_{n_1 m_1} \langle q_1 | n_1 \rangle \langle n_1 | \hat{f}(t) | m_1 \rangle \langle m_1 | q_1' \rangle, \tag{113}$$

where $\phi_n(q)$ are the eigenstates of the harmonic oscillator, given by

$$\phi_n(q) = \langle q | n \rangle \equiv \left(\frac{M_1 \omega_1}{\pi \hbar} \right)^{1/4} \left(\frac{1}{2^n n!} \right)^{1/2} H_n(\xi) e^{-(\xi^2/2)}, \tag{114}$$

with $H_n(\xi)$ being a Hermite polynomial of degree n and $\xi \equiv (M_1 \omega_1 / \hbar)^{1/2} q$, the Eq. (112) takes the following form [29]:

$$i\hbar \frac{\partial}{\partial t} f(q_1, q_1', t) = \left[-\frac{\hbar^2}{2M_1} \left(\frac{\partial^2}{\partial q_1^2} - \frac{\partial^2}{\partial q_1'^2} \right) + \frac{M_1 \tilde{\omega}_1^2}{2} \right.$$

$$\times (q_1^2 - q_1'^2) \Big] f(q_1, q_1', t) - i\lambda^2 A_2$$

$$\times (q_1 - q_1')^2 f(q_1, q_1', t) + i\hbar^2 \lambda^2 B_2$$

$$\times \left(\frac{\partial}{\partial q_1} + \frac{\partial}{\partial q_1'} \right)^2 f(q_1, q_1', t) - i\hbar \lambda^2 \Gamma_2$$

$$\times (q_1 - q_1') \left(\frac{\partial}{\partial q_1} - \frac{\partial}{\partial q_1'} \right) f(q_1, q_1', t)$$

$$+ i\hbar \lambda^2 \Gamma_2$$

$$\times \left(\frac{\partial}{\partial q_1} + \frac{\partial}{\partial q_1'} \right) (q_1 + q_1') f(q_1, q_1', t). \tag{115}$$

Here the constant coefficients A_2, B_2, and Γ_2 are given by

$$A_2 \equiv \frac{M_1 \omega_1}{2} \int dk \, \gamma_k \left(\langle n_k \rangle_0 + \frac{1}{2} \right), \tag{116}$$

$$B_2 \equiv \frac{1}{2M_1 \omega_1} \int dk \, \gamma_k \left(\langle n_k \rangle_0 + \frac{1}{2} \right), \tag{117}$$

$$\Gamma_2 \equiv \frac{1}{2} \int dk \frac{\gamma_k}{2} = \frac{\gamma}{4}, \tag{118}$$

where the subscript 2 in A_2, B_2, and Γ_2 indicate the second-order contribution in λ, and we have explicitly indicated $\hbar$ and the mass M_1 of the harmonic oscillator.

Finally, in the Wigner representation defined by

$$f^W(Q_1, P_1, t) \equiv \frac{1}{2\pi\hbar} \int dr_1 e^{iP_1 r_1/\hbar} f\left(Q_1 + \frac{r_1}{2}, Q_1 - \frac{r_1}{2}, t \right) \tag{119}$$

our master equation (115) takes the form

$$\frac{\partial}{\partial t} f^W(Q_1, P_1, t) = \left(-\frac{1}{M_1} P_1 \frac{\partial}{\partial Q_1} + M_1 \tilde{\omega}_1^2 Q_1 \frac{\partial}{\partial P_1} \right.$$

$$+ \lambda^2 \hbar A_2 \frac{\partial^2}{\partial P_1^2} + \lambda^2 \hbar B_2 \frac{\partial^2}{\partial Q_1^2}$$

$$+ 2\lambda^2 \Gamma_2 \frac{\partial}{\partial P_1} P_1$$

$$\left. + 2\lambda^2 \Gamma_2 \frac{\partial}{\partial Q_1} Q_1 \right) f^W(Q_1, P_1, t), \tag{120}$$

where P_1 and $Q_1 = (q_1 + q_1')/2$ are the Brownian particle momentum and average position, respectively.

In Eq. (120) the terms associated with the coefficients A_2 and B_2 describe the diffusion process in variables P_1 and Q_1, respectively. These terms are responsible for the decoherence phenomena. Indeed, in Eq. (115) one can see that the term associated with A_2 leads to the decoherence of the off-diagonal elements of the reduced density matrix in the coordinate representation, and the decoherence time t_{dq} is given by

$$t_{dq} \sim \frac{\hbar}{\lambda^2 A_2 (q_1 - q_1')^2}. \quad (121)$$

Similarly, the decoherence time t_{dp} in the momentum representation is given by

$$t_{dp} \sim \frac{\hbar}{\lambda^2 B_2 (p_1 - p_1')^2}. \quad (122)$$

The larger the $|q_1 - q_1'|$ or $|p_1 - p_1'|$, the shorter the decoherence time t_d. However, there is a lower bound of t_d, below which formulas (121) and (122) are not applicable. Indeed, for times much shorter than the relaxation time scale one cannot approximate $P^{(\nu_1)}\rho(t)$ by only the privileged component, as in Eq. (97). In the following section, we will evaluate a contribution from the nonprivileged component in Eq. (90) for times much shorter than the relaxation time scale.

To consistently take the classical limit $\hbar \to 0$, one has to keep in mind that in the RHS of Eq. (120) the decoherence rate constants $A_2 \sim \hbar^{-1}$ and $B_2 \sim \hbar^{-1}$. Then, in this limit we recover the usual kinetic equation for the reduced probability distribution function of a classical particle weakly coupled to the classical field. Note that in the classical limit, in the RHS of the kinetic equation we still have the terms proportional to the second-order differential operators of position $\partial^2/\partial Q_1^2$ and momentum $\partial^2/\partial P_1^2$ of the classical particle. Therefore, quantum decoherence or collapse of wave function is not a unique attribute of quantum mechanics and corresponds to the well-known diffusion processes in classical mechanics.

V. QUANTUM ZENO TIME

In this section we will show that the non-Markovian effect coming from the nonprivileged components in Eq. (90) gives a non-negligible contribution for a short time scale, $t \ll \lambda^{-2}$, and it dies out following a power law in a time scale of the order $1/\omega_1 \ll \lambda^{-2}$. Therefore, $1/\omega_1$ gives a time scale of transition of the evolution from the non-Markovian regime to the Markovian regime. This transition time scale is called quantum Zeno time [26]. Only after the Zeno time does the Markov equation (102) start making sense. Hence, quantum Zeno time serves as a lower bound for the decoherence time [29].

To estimate the Zeno time, we first note that to generate the correlation subspace $P^{(\nu_1' \nu_k \cdots \nu_l)}$ from the $P^{(\nu_1)}$ subspace, we need at least $|\nu_k| + \cdots + |\nu_l|$ successive interactions, i.e.,

$$P^{(\nu_1)} \mathbf{C}^{(\nu_1' \nu_k \cdots \nu_l)} P^{(\nu_1' \nu_k \cdots \nu_l)} \sim P^{(\nu_1' \nu_k \cdots \nu_l)} \mathbf{D}^{(\nu_1' \nu_k \cdots \nu_l)} P^{(\nu_1)}$$

$$\sim O(\lambda^{|\nu_k| + \cdots + |\nu_l|}). \quad (123)$$

Hence, the dominant contribution in a short time scale in expression (90) comes from $\Pi^{(\mu_2)}$ subspace for $\lambda \ll 1$, where $\mu_2 \equiv (\mu_1' \pm 1)$. In the short time scale $t \ll \lambda^{-2}$, we can approximate the collision operator $\theta^{(\nu_1', \nu_k)}$ as [cf. Eq. (93)]

$$\theta^{(\nu_1', \nu_k)} \approx \theta_0^{(\nu_1', \nu_k)} = (\nu_1' \omega_1 + \nu_k \omega_k) P^{(\nu_1', \nu_k)}. \quad (124)$$

Then, we have

$$\langle n_1 | \mathrm{Tr}_F [P^{(\nu_1)} \rho_{np}(t)] | m_1 \rangle$$

$$\approx \lambda^2 \sum_{\nu_1'} \sum_k \sum_{\nu_k = \pm 1} \langle n_1 | \mathrm{Tr}_F [P^{(\nu_1)} \mathbf{C}_1^{(\nu_1', \nu_k)}$$

$$\times \exp[-i(\nu_1' \omega_1 + \nu_k \omega_k) t] \mathbf{D}_1^{(\nu_1', \nu_k)} P^{(\nu_1)} \rho(0)] | m_1 \rangle. \quad (125)$$

The calculation of the RHS of expression (125) is straightforward. We shall present this calculation in Appendix B. Here we display only the final result,

$$\langle n_1 | \mathrm{Tr}_F [P^{(\nu_1)} \rho_{np}(t)] | m_1 \rangle$$

$$\approx \lambda^2 \int dk |v_k|^2 \Big\{ [(\langle n_k \rangle_0 + 1)(n_1 + m_1) f_{n_1, m_1}(0)$$

$$+ \langle n_k \rangle_0 (n_1 + m_1 + 2) f_{n_1, m_1}(0)$$

$$+ \langle n_k \rangle_0 \sqrt{n_1 m_1} f_{n_1-1, m_1-1}(0) + (\langle n_k \rangle_0$$

$$+ 1) \sqrt{(n_1+1)(m_1+1)} f_{n_1+1, m_1+1}(0)]$$

$$\times \left(\frac{\exp[-i(\omega_k - \omega_1)t]}{(\omega_k - \omega_1 - i\epsilon)^2} + \mathrm{c.c.} \right)$$

$$+ [(\langle n_k \rangle_0 + 1)(n_1 + m_1 + 2) f_{n_1, m_1}(0) + \langle n_k \rangle_0$$

$$\times (n_1 + m_1) f_{n_1, m_1}(0) + (\langle n_k \rangle_0 + 1) \sqrt{n_1 m_1} f_{n_1-1, m_1-1}(0)$$

$$+ \langle n_k \rangle_0 \sqrt{(n_1+1)(m_1+1)} f_{n_1+1, m_1+1}(0)]$$

$$\times \left(\frac{\exp[-i(\omega_k + \omega_1)t]}{(\omega_k + \omega_1 - i\epsilon)^2} + \mathrm{c.c.} \right) \Big\} e^{-i\nu_1 \omega_1 t}. \quad (126)$$

As a function of $\omega_k = |k|$ the first and second terms in the integrand in expression (126) have a second-order poles in the upper and lower half of the complex plane of ω_k. Hence, in the $\Pi^{(\nu_1' \nu_k)}$ subspace, the resonance poles do not contribute for $t > 0$. Indeed, these resonance effects have already been taken into account in the $\Pi^{(\nu_1')}$ subspace. In the subdynamics theory, analytic continuation of the denominators has been carefully chosen to avoid double counting of the resonance effects.

As a working example, let us evaluate a part of the first term in Eq. (126), given by

$$I(t) = \lambda^2 (n_1 + m_1) f_{n_1,m_1}(0) \int dk |v_k|^2 (\langle n_k \rangle_0 + 1)$$

$$\times \frac{\exp[-i(\omega_k - \omega_1)t]}{(\omega_k - \omega_1 - i\epsilon)^2}. \tag{127}$$

Assuming that $\langle n_k \rangle_0 = \exp(-\beta\omega_k)$ with $\beta = (k_B T)^{-1}$ and (cf. [33])

$$|v_k|^2 = \frac{\omega_k}{[1 + (\omega_k/M)^2]^4}, \tag{128}$$

where M is the cutoff frequency. Then, we have

$$I(t) \equiv \lambda^2 (n_1 + m_1) f_{n_1,m_1}(0) \int_{-\infty}^{\infty} dk \frac{|v_k|^2 (\langle n_k \rangle_0 + 1)}{(|k| - \omega_1 - i\epsilon)^2}$$

$$\times \exp[-i(|k| - \omega_1)t]$$

$$= 2\lambda^2 (n_1 + m_1) f_{n_1,m_1}(0) \int_0^{\infty}$$

$$\times d\omega \frac{|v_\omega|^2 (e^{-\beta\omega} + 1)}{(\omega - \omega_1 - i\epsilon)^2} e^{-i(\omega - \omega_1)t}. \tag{129}$$

For $t > 0$, we deform the contour of integration in the lower-half plane of ω. A contour rotation $\omega = -iy$ with $y = \xi/t$ allows us to evaluate the integration in Eq. (129) as

$$I(t) = -2i\lambda^2 (n_1 + m_1) f_{n_1,m_1}(0) e^{i\omega_1 t} t$$

$$\times \int_0^{\infty} d\xi \frac{|v_{-i(\xi/t)}|^2}{(\xi - i\omega_1 t)^2} e^{-\xi} (1 + e^{i(\beta\xi/t)}). \tag{130}$$

Due to the factor $\exp(-\xi)$, integral over ξ contributes primarily for values of $\xi \sim 1$. Hence, for $1 \ll \omega_1 t \ll \omega_1 \lambda^{-2}$ integration in Eq. (130) with Eq. (128) yields

$$I(t) \approx -4\lambda^2 \frac{e^{i\omega_1 t}}{(\omega_1 t)^2} (n_1 + m_1) f_{n_1,m_1}(0) \int_0^{\infty} d\xi \xi e^{-\xi}$$

$$= -4\lambda^2 \frac{e^{i\omega_1 t}}{(\omega_1 t)^2} (n_1 + m_1) f_{n_1,m_1}(0), \tag{131}$$

where we have approximated $\exp(i\beta\xi/t) \approx 1$ for $\xi/t \ll 1$. A similar estimation shows that all other terms decay as $(\omega_1 t)^{-2}$.

This result shows that after a time scale of $1/\omega_1$, the memory effect described by the nonprivileged components in expression (90) becomes negligible as compared with the contribution from the privileged component that obeys the Markov kinetic equation (102). A deviation from the Markov evolution is known as the quantum Zeno effect [26]. Equation (131) shows that the Zeno time is given as $1/\omega_1$. After this time scale, the decoherence terms with coefficients A_2 and B_2 in Eq. (120) start to give non-negligible contributions. Therefore, we see that the Zeno time gives a lower bound for the decoherence time.

VI. DISCUSSION

As an application of complex spectral representations of the Liouvillian, we have investigated the motion of a charged harmonic oscillator coupled to a field. For a weakly coupled case, we have analyzed a short time memory effect that dies out in quantum Zeno time of the order $1/\omega_1$, and derived kinetic equations (101) and (102) that are valid in the long time scale of order of the relaxation time. As far as the final form of these kinetic equations are concerned, any standard approach based on the $\lambda^2 t$ approximation (that recovers the well-known Pauli master equation) leads to the same kinetic equations.

An additional element is the analysis of the memory effect (i.e., the quantum Zeno effect discussed in Sec. V) associated with non-Markovianity in a short time scale of the order $1/\omega_1$. For a time scale shorter than the Zeno time, the Markovian kinetic equation is not a good approximation to describe the evolution of the system and the expressions (121) and (122) for decoherence times lose their meaning.

In this paper, we have restricted our consideration to only the dominant contributions in the expansion of the collision operator in terms of the coupling constant. However, we should emphasize that our estimation of the memory effect in Sec. V goes beyond the $\lambda^2 t$ approximation, that is, the lowest-order contribution in the Markovian regime. Except for a rigorous derivation for the generalized master equation, estimation of contributions from the memory effects, that are corrections to the $\lambda^2 t$ approximation, is still a controversial subject [19–21]. Our complex spectral representation of the Liouvillian offers a systematic way of estimating contributions from the memory effects even for a higher-order contribution with respect to the coupling constant.

As mentioned in the introduction, in a typical setting for the decoherence problem (or quantum Brownian motion), one usually assumes a particle (a subsystem) embedded in a field, that is, in thermodynamic equilibrium [see Eq. (87)]. In this case, the so-called collapse of wave functions of the subsystem is trivially a contamination of the subsystem by the mixed nature of the surrounding field. In contrast, our complex spectral representation of the Liouvillian is not restricted to near equilibrium, and is applicable to arbitrary initial conditions including the pure state (88), which is far from equilibrium and satisfies the thermodynamic condition. Our result in Sec. IV shows that starting with this pure initial condition, the subsystem evolves in time obeying the diffusion equation (120), while a field component keeps purity as its distribution remains invariant in the thermodynamic situation [see Eq. (111)]. Therefore, our result presents a striking nontrivial example of a dynamical collapse of wave functions in the thermodynamic limit. It should be emphasized again that this transition of pure to mixture occurs because the evolution takes place outside Hilbert space.

Before closing our paper, let us give some remarks on other approaches to the problem of quantum decoherence. The usual phenomenological approach to this problem is to start with the Pauli master equation that is a Markovian equation [9]. Various examples of the estimation of the decoherence time scale for a particle in a gas in thermal equi-

librium can be found in Refs. [34,35] (see also a book by Giulini *et al.* [9]). Applying their estimates to several sizes of the tagged particle, these authors obtained decoherence times at room temperature of, for example, 10^{-18} s for a large molecule of size 10^{-6} cm, 10^{-30} s for a dust particle with size 10^{-3} cm. Hence, it seems that the decoherence time after which a classical world appears may be extremely short for both mesoscopically and macroscopically sized materials.

However, these estimations are meaningless, since the estimated time scale is much shorter than the relaxation time where the Markovian process is not a good description of the evolution of the system. For instance, the relaxation time scale for a gas is typically 10^{-6} s at room temperature. As we have shown in this paper, for a short time scale memory effects to the initial condition remain. Because the diffusion process is a result of resonance interactions, it takes time to build up this process.

Concerning the decoherence time scale, more confusion in the interpretation has resulted from the studies of the so-called "exactly solvable model" discussed by Unruh and Zurek [8]. They have considered the same type of system discussed in this paper. The resulting master equation leads to a dissipative term even in the limit $t \rightarrow 0$. Hence, the dissipation seems to start instantaneously, and so does the decoherence. However, their result is inconsistent with dynamics. Indeed, the generalized master equation (1) shows that dissipation disappears in the limit $t \rightarrow 0$.[5]

Later, this inconsistency has been noticed by Hu *et al.* [23,24]. By applying the path-integral method initiated by Caldeira and Leggett [22] for quantum Brownian motion, they have obtained another form of the kinetic equation, in which dissipation disappears in the limit $t \rightarrow 0$. However, as we now show, their kinetic equation still contradicts with dynamics, since it does not consistently describe a spontaneous decay of an excited harmonic oscillator.

The main difficulty in their kinetic equation comes from their so-called "anomalous diffusion" term. Note that our kinetic equation (115) does not have the anomalous diffusion term. As shown in Appendix C, their kinetic equation reduces to the following Markovian equation in the $\lambda^2 t$ approximation

$$i\hbar \frac{\partial}{\partial t} f(q_1, q_1', t) = \left[-\frac{\hbar^2}{2M_1}\left(\frac{\partial^2}{\partial q_1^2} - \frac{\partial^2}{\partial q_1'^2} \right) + \frac{M_1 \tilde{\omega}_1^2}{2}(q_1^2 \right.$$
$$\left. - q_1'^2) \right] f(q_1, q_1', t) - i\hbar \lambda^2 \Gamma_2 (q_1 - q_1')$$
$$\times \left(\frac{\partial}{\partial q_1} - \frac{\partial}{\partial q_1'} \right) f(q_1, q_1', t) - i\lambda^2 A_2 (q_1$$

―――――
[5]As pointed out by Hu *et al.* [23], the singular nature of the interaction (such as the ultraviolet divergence) may lead to a nonvanishing contribution to the interaction term in Eq. (1) in the limit $t \rightarrow 0$. However, since the ultraviolet singularity is irrelevant to irreversibility, the nonvanishing term (if it exists) should not be related to dissipation.

$$- q_1')^2 f(q_1, q_1', t) + i\lambda^2 \hbar D_{an}(q_1 - q_1')$$
$$\times \left(\frac{\partial}{\partial q_1} + \frac{\partial}{\partial q_1'} \right) f(q_1, q_1', t), \qquad (132)$$

which is different from our Eq. (115). The last term in the RHS of Eq. (132) is their anomalous diffusion term. We note that because of this term, their kinetic equation does not lead to the Pauli-type kinetic equations (108) and (107).

We now show that our kinetic equation (115) leads to a consistent description of dynamics, while this is not the case for Eq. (132). We first consider the zero temperature case $T = 0$. Then we have $\langle n_k \rangle_0 = 0$ in Eqs. (116) and (117). This corresponds to the nonthermodynamic situation (27a), and one can exactly solve the evolution of the system by the Bogoljubov transformation given in Appendix D. Let us assume that the initial condition is given by

$$\rho(0) = |\Psi(0)\rangle\langle\Psi(0)| = |1_p, 0_k\rangle\langle 1_p, 0_k| \qquad (133)$$

with

$$|\Psi(0)\rangle = |1_p, 0_k\rangle \equiv a_1^\dagger |0_p, 0_k\rangle, \qquad (134)$$

where 0_p and 0_k denote the vacuum of particle and the field, respectively. This corresponds to the well-known Friedrichs model (for detailed studies of this model see, for example, Refs. [36–39]). Let us then consider the $\lambda^2 t$ limit, i.e.,

$$\lambda \rightarrow 0, \quad t \rightarrow +\infty \quad \text{with} \quad \lambda^2 t = \text{finite}. \qquad (135)$$

We should distinguish the concept of the $\lambda^2 t$ *limit* and the $\lambda^2 t$ *approximation*. The expressions obtained in $\lambda^2 t$ limit are *exact* in the limit $t \rightarrow \infty$ for Eq. (135), while being *approximations* when we apply these expressions for a finite time scale of the order λ^{-2} with finite λ.

As shown in Appendix D, the Bogoliubov transformation leads to the *exact solution* for the reduced particle distribution function in the $\lambda^2 t$ limit,

$$f(q_1, q_1', t) \rightarrow (1 - e^{-\lambda^2 \gamma t}) \phi_0(q_1) \phi_0^*(q_1')$$
$$+ e^{-\lambda^2 \gamma t} \phi_1(q_1) \phi_1^*(q_1'), \qquad (136)$$

where $\phi_0(q_1)$ and $\phi_1(q_1)$ are the ground and first excited state of the particle, respectively, given in Eq. (114). One can verify that Eq. (136) is indeed the solution of our kinetic equation, by substituting Eq. (136) in both sides of Eq. (115). However, this is not the case for Eq. (132). Therefore, the kinetic equation of Hu *et al.* contradicts with the exact time evolution of the system for the zero temperature case.

We note that for an arbitrary temperature of the field, the anomalous diffusion term in Eq. (132) vanishes at the point $q_1' = q_1$, while the corresponding terms with coefficients B_2 and Γ_2 in our kinetic equation do not vanish. Therefore, the discrepancy between the kinetic equation of Hu *et al.* and the underlying dynamics remains even for the nonzero temperature case.
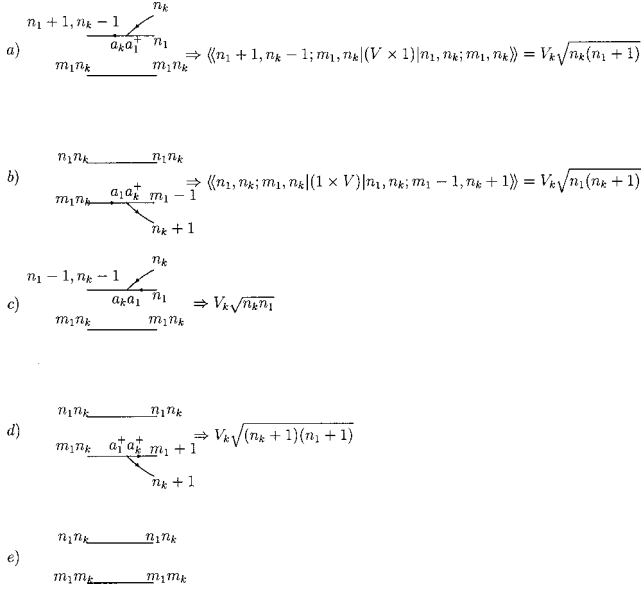
FIG. 1. Examples of typical vertices for real transitions (a) and (b), virtual transitions (c) and (d), and the propagator (e).

VII. CONCLUSION

In summary we have shown that quantum decoherence is a phenomenon taking place outside Hilbert space in the thermodynamic limit. Our theory can be applied to the case when initial conditions are far from equilibrium, such as a pure state. We have demonstrated that transition from the pure state to a mixed state (collapse of wave functions) is dynamically possible in the thermodynamic limit. We have shown that quantum decoherence is not a unique attribute of quantum mechanics, as in the classical limit the decoherence terms reduce to the diffusion terms. We have also shown that quantum Zeno time serves as a lower bound for the decoherent time. The estimated decoherence time is much longer than the time reported in literature.

APPENDIX A: DERIVATION OF THE MASTER EQUATIONS

In this appendix we outline steps in the calculation of the collision operator that yield expressions in the RHS of the kinetic equations (101) and (102). It is convenient to use the correlation space diagrams to describe each process contributing to the collision operator. Matrix elements associated

with interaction $V \times 1$ and $-1 \times V$ are represented by the vertex in Fig. 1 (fragments a, b, c, and d), and are given by

$$\langle\langle \alpha; \beta | [V \times 1] | \alpha'; \beta' \rangle\rangle = V_{\alpha \alpha'} \delta_{\beta' \beta},$$

$$-\langle\langle \alpha; \beta | [1 \times V] | \alpha'; \beta' \rangle\rangle$$

$$= -\delta_{\alpha \alpha'} V_{\beta' \beta}. \tag{A1}$$

In the following expressions we use the abbreviated notation for the state

$$|n_1, n_k; m_1, n_k\rangle\rangle \equiv |n_1, n_k, \{n_F\}^{N-1}; m_1, n_k, \{n_F\}^{N-1}\rangle\rangle. \tag{A2}$$

Each diagram consists of the upper part corresponding to $(V \times 1)$ and the lower part corresponding to $-(1 \times V)$. Straight lines correspond to the particle, while curved lines correspond to the field. Each vertex contains $a_1^\dagger a_k$, $a_k^\dagger a_1$, $a_1^\dagger a_k^\dagger$, $a_1 a_k$. For the interaction, the direction of reading is denoted by arrows, and is from left to right on the lower line and from right to left on the upper line. The arrow oriented to the vertex corresponds to the annihilation operator a, while the arrow coming out of the vertex corresponds to the creation operator $a^\dagger$.

The propagator is given by

$$\langle\langle n_1, n_k; m_1, m_k | \frac{-1}{H_0 \times 1 - 1 \times H_0 - z} | n_1, n_k; m_1, m_k \rangle\rangle$$

$$= \frac{-1}{\mu_1 \omega_1 + \mu_k \omega_k - z}, \tag{A3}$$

with $\mu_1 \equiv n_1 - m_1$ and $\mu_k \equiv n_k - m_k$. The propagator is represented by the diagram e in Fig. 1.

We calculate the matrix element of the collision operator (76) in the number representation

$$\langle\langle n_1, n_k; m_1, m_k | \psi_2^{(\nu_1)}(w^{(\nu_1)} + i\epsilon) | n_1', n_k'; m_1', m_k' \rangle\rangle$$

$$= \langle\langle n_1, n_k; m_1, m_k | P^{(\nu_1)} L_0 P^{(\nu_1)} | n_1', n_k'; m_1', m_k' \rangle\rangle$$

$$+ \left\langle \langle n_1, n_k; m_1, m_k \left| P^{(\nu_1)} L_V Q^{(\nu_1)} \frac{-1}{L_0 - w^{(\nu_1)} - i\epsilon} \right. \right.$$

$$\left. \times Q^{(\nu_1)} L_V P^{(\nu_1)} | n_1', n_k'; m_1', m_k' \rangle \right\rangle. \tag{A4}$$

In Eq. (A4) the first term corresponds to free propagation without interaction, the second term represents interaction. Calculating the free part in Eq. (A2), we obtain

$$\langle\langle n_1, n_k; m_1, m_k | P^{(\nu_1)} L_0 P^{(\nu_1)} | n_1', n_k'; m_1', m_k' \rangle\rangle$$

$$= \omega_1 (n_1 - m_1) \delta_{n_1', n_1} \delta_{m_1', m_1} \delta_{n_k', n_k} \delta_{m_k', m_k}. \tag{A5}$$

The diagrams corresponding to the second term in Eq. (A4) are presented in Figs. 2 and 3, where we have collected the processes coming from the "real" transitions in Fig. 2, while the processes from the "virtual" transitions in Fig. 3. As we will see, the virtual processes contribute only to the frequency shift for the renormalized particle in the $\lambda^2 t$ ap-
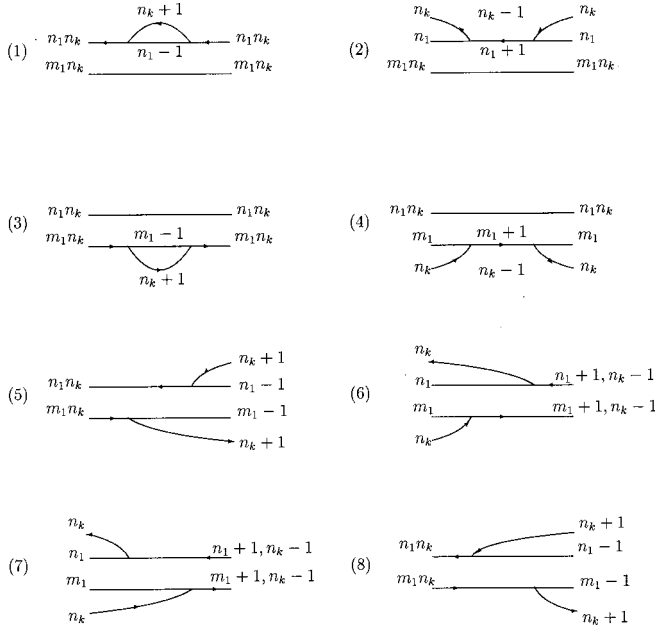
FIG. 2. The second-order diagrams with "real" transitions in the correlation space.

proximation. As an example, we first give the explicit derivation of a contribution from one typical diagram [say, diagram (5) with real transitions in Fig. 2]. The propagator part (A2) of diagram (5) corresponds to

$$(L_0 - w^{(\nu_1)} - i\epsilon) \Rightarrow [[(n_1 - m_1 + 1)\omega_1 + (n_k - n_k - 1)\omega_k]$$
$$- (n_1 - m_1)\omega_1 - i\epsilon] = \omega_1 - \omega_k - i\epsilon. \quad (A6)$$

The vertices are calculated as
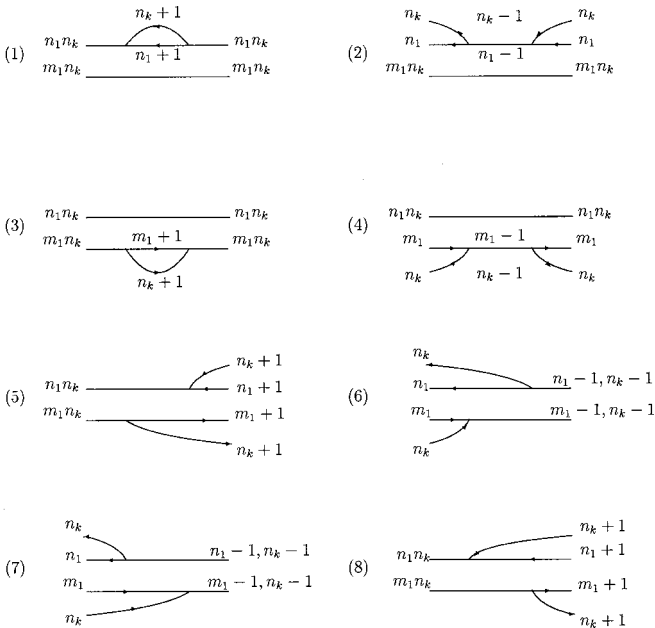
FIG. 3. The second-order diagrams with "virtual" transitions in the correlation space.

$$-\langle\langle n_1,n_k;m_1,m_k|[1 \times V_k a_1 a_k^+]|n_1,n_k;m_1-1,m_k+1\rangle\rangle$$
$$= -V_k\sqrt{m_1(n_k+1)}, \quad (A7)$$

$$\langle\langle n_1,n_k;m_1-1,n_k+1|[V_k a_1^+ a_k \times 1]|n_1-1,n_k+1;m_1-1,n_k$$
$$+1\rangle\rangle = V_k\sqrt{n_1(n_k+1)}. \quad (A8)$$

Then, using the expression (53) for the collision operator, we obtain the following contribution to the interaction part in Eq. (A4) from diagram (5):

$$\sum_k (-V_k)\sqrt{m_1(n_k+1)} \frac{-1}{\omega_1 - \omega_k - i\epsilon} V_k\sqrt{n_1(n_k+1)}\rho(n_1$$
$$-1,n_k+1;m_1-1,n_k+1,t)$$

$$= \sum_k |V_k|^2(n_k+1)\sqrt{n_1 m_1} \frac{1}{\omega_1 - \omega_k - i\epsilon}$$
$$\times \rho(n_1-1,n_k+1;m_1-1,n_k+1,t). \quad (A9)$$

One can evaluate contributions from the remaining diagrams in a similar way. Collecting all intraction part contributions to Eq. (A4) from diagrams (1)–(4) and (6)–(8) with real transitions (Fig. 2), we get the following expressions:

$$(1) \quad \sum_k |V_k|^2 n_1(n_k+1) \frac{1}{\omega_k - \omega_1 - i\epsilon}\rho(n_1,n_k;m_1,n_k,t),$$

$$(2) \quad \sum_k |V_k|^2 n_k(n_1+1) \frac{1}{\omega_1 - \omega_k - i\epsilon}\rho(n_1,n_k;m_1,n_k,t),$$

$$(3) \quad \sum_k |V_k|^2 m_1(n_k+1) \frac{1}{\omega_1 - \omega_k - i\epsilon}\rho(n_1,n_k;m_1,n_k,t),$$

$$(4) \quad \sum_k |V_k|^2 n_k(m_1+1) \frac{1}{\omega_k - \omega_1 - i\epsilon}\rho(n_1,n_k;m_1,n_k,t),$$

$$(6) \quad \sum_k |V_k|^2 n_k\sqrt{(n_1+1)(m_1+1)} \frac{1}{\omega_k - \omega_1 - i\epsilon}$$
$$\times \rho(n_1+1,n_k-1;m_1+1,n_k-1,t),$$

$$(7) \quad \sum_k |V_k|^2 n_k\sqrt{(n_1+1)(m_1+1)} \frac{1}{\omega_1 - \omega_k - i\epsilon}$$
$$\times \rho(n_1+1,n_k-1;m_1+1,n_k-1,t),$$

$$(8) \quad \sum_k |V_k|^2 \sqrt{n_1 m_1}(n_k+1) \frac{1}{\omega_k - \omega_1 - i\epsilon}$$
$$\times \rho(n_1-1,n_k+1;m_1-1,n_k+1,t). \quad (A10)$$

As a next example, we illustrate the calculation of diagram (1) with virtual processes in Fig. 3. The propagator part in diagram (1) corresponds to

$$(\omega^{(\nu_1)} + i\epsilon - L_0) \Rightarrow (n_1 - m_1) + i\epsilon - [(n_1 + 1 - m_1)\omega_1$$

$$-(n_k + 1 - n_k)\omega_k] = i\epsilon - \omega_1 - \omega_k. \tag{A11}$$

The vertices are

$$\langle\langle n_1 + 1, n_k + 1; m_1, m_k | [V_k a_k^\dagger a_1^\dagger \times 1] | n_1, n_k; m_1, m_k \rangle\rangle$$

$$= V_k \sqrt{(n_1 + 1)(n_k + 1)},$$

$$\langle\langle n_1, n_k; m_1, m_k | [V_k a_k a_1 \times 1] | n_1 + 1, n_k + 1; m_1, m_k \rangle\rangle$$

$$= V_k \sqrt{(n_1 + 1)(n_k + 1)}. \tag{A12}$$

Then, for diagram (1) with virtual processes in Fig. 3, we obtain

$$\sum_k V_k \sqrt{(n_1 + 1)(n_k + 1)} \frac{-1}{i\epsilon - \omega_1 - \omega_k}$$

$$\times V_k \sqrt{(n_1 + 1)(n_k + 1)} \rho(n_1, n_k; m_1, n_k, t) f$$

$$= \sum_k |V_k|^2 (n_k + 1)(n_1 + 1)$$

$$\times \frac{1}{\omega_1 + \omega_k - i\epsilon} \rho(n_1, n_k; m_1, n_k, t). \tag{A13}$$

Contributions to Eq. (A4) from diagrams (2)–(8) with virtual processes are

$$(2) \ \sum_k |V_k|^2 n_k n_1 \frac{1}{-i\epsilon - \omega_1 - \omega_k} \rho(n_1, n_k; m_1, n_k, t),$$

$$(3) \ \sum_k |V_k|^2 (m_1 + 1)(n_k + 1) \frac{1}{-i\epsilon - \omega_1 - \omega_k}$$

$$\times \rho(n_1, n_k; m_1, n_k, t),$$

$$(4) \ \sum_k |V_k|^2 n_k m_1 \frac{1}{-i\epsilon + \omega_1 + \omega_k} \rho(n_1, n_k; m_1, n_k, t),$$

$$(5) \ \sum_k |V_k|^2 \sqrt{n_1 m_1} n_k \frac{1}{-i\epsilon + \omega_1 + \omega_k}$$

$$\times \rho(n_1 - 1, n_k - 1; m_1 - 1, n_k - 1, t),$$

$$(6) \ \sum_k |V_k|^2 (n_k + 1) \sqrt{(n_1 + 1)(m_1 + 1)} \frac{1}{-i\epsilon - \omega_1 - \omega_k}$$

$$\times \rho(n_1 + 1, n_k + 1; m_1 + 1, n_k + 1, t),$$

$$(7) \sum_k |V_k|^2 (n_k + 1) \sqrt{(n_1 + 1)(m_1 + 1)}$$

$$\times \frac{1}{-i\epsilon + \omega_1 + \omega_k} \rho(n_1 + 1, n_k + 1; m_1 + 1, n_k + 1, t),$$

$$(8) \ \sum_k |V_k|^2 \sqrt{n_1 m_1} n_k \frac{1}{-i\epsilon - \omega_1 - \omega_k}$$

$$\times \rho(n_1 - 1, n_k - 1; m_1 - 1, n_k - 1, t), \tag{A14}$$

Using the relation for $\epsilon \rightarrow 0+$,

$$\frac{1}{\omega \pm i\epsilon} \rightarrow P\left(\frac{1}{\omega}\right) \mp \pi i \delta(\omega), \tag{A15}$$

we notice that in contributions (A12) and (A13) from diagrams with virtual transitions, there appears the δ function in the form $\delta(\omega_1 + \omega_k)$ that gives a vanishing contribution. Contributions (A9) and (A10) from diagrams with real transitions, contain the δ function of the form $\delta(\omega_1 - \omega_k)$ that gives a nonvanishing dissipative contribution at the "resonance point" $\omega_k = \omega_1$. The principal part of the propagator gives the nonvanishing contribution to the frequency shift.

Summing over all contributions, we obtain the master equation for the total density matrix for $t \sim \lambda^{-2}$

$$i\frac{\partial}{\partial t}\rho(n_1, n_k; m_1, n_k)$$

$$= \langle\langle n_1, n_k; m_1, n_k | \psi_2^{(\nu_1)}(w^{(\nu_1)} + i\epsilon) | \rho \rangle\rangle$$

$$= (n_1 - m_1)\left[\omega_1 - \lambda^2 \sum_k |V_k|^2 P\left(\frac{2\omega_1}{\omega_1^2 - \omega_k^2}\right)\right]$$

$$\times \rho(n_1, n_k; m_1, n_k, t) + \lambda^2 2\pi i \sum_k |V_k|^2 \delta(\omega_k - \omega_1)$$

$$\times \left[\sqrt{n_1 m_1}(n_k + 1)\rho(n_1 - 1, n_k + 1; m_1 - 1, n_k + 1, t)\right.$$

$$-\frac{1}{2}(n_1 + m_1 + 2n_k + 2n_1 n_k + 2m_1 n_k)$$

$$\times \rho(n_1, n_k; m_1, n_k, t) + n_k \sqrt{(n_1 + 1)(m_1 + 1)}$$

$$\left. \times \rho(n_1 + 1, n_k - 1; m_1 + 1, n_k - 1, t)\right]. \tag{A16}$$

By reducing Eq. (A16) as in Eqs.(85) and (86), and taking the continuous spectrum limit $\Omega \rightarrow \infty$, we obtain the kinetic equations (101) and (102) of the main text, where we have used the factorizability property (81), which persists for any time $t > 0$ [17].

APPENDIX B: DERIVATION OF EQ. (121)

In this appendix we evaluate contributions from the non-privileged component (92) to the time evolution of the density matrix (90) and derive Eq. (126). We can use the same diagrams presented in Appendix A, but with a different correspondence to the mathematical expression. For the non-privileged component we associate the expression (92) to the diagram. For the interaction part, the correspondence is the same as the one presented in Appendix A. In the propagator, care must be taken in the analytic continuation to be consistent with the evolution oriented to $t > 0$. Because the subspace $(\nu_1' \nu_k')$ is more correlated than the subspace (ν_1), we have to associate the creation operator with the propagator [6]

$$P^{(\nu_1)}C_1^{(\nu_1' \nu_k')}P^{(\nu_1' \nu_k')} \Rightarrow \frac{-1}{\nu_1 \omega_1 - (\nu_1' \omega_1 + \nu_k' \omega_k) + i\epsilon}, \tag{B1}$$

while the destruction operator with the propagator

$$P^{(\nu_1' \nu_k')} D_1^{(\nu_1', \nu_k')} P^{(\nu_1)} \Rightarrow \frac{+1}{(\nu_1' \omega_1 + \nu_k' \omega_k) - i\epsilon - \nu_1 \omega_1}. \quad \text{(B2)}$$

Using the correspondence for each diagram in Fig. 2, we obtain the following expressions [by omitting a common factor $\exp(-i\nu_1\omega_1 t)$]:

(1) $\sum_k |V_k|^2 n_1(n_k+1)\exp[-i(\omega_k-\omega_1)t]$

$$\times \frac{1}{(\omega_k-\omega_1-i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(2) $\sum_k |V_k|^2 n_k(n_1+1)\exp[-i(\omega_1-\omega_k)t]$

$$\times \frac{1}{(\omega_1-\omega_k-i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(3) $\sum_k |V_k|^2 m_1(n_k+1)\exp[-i(\omega_1-\omega_k)t]$

$$\times \frac{1}{(\omega_1-\omega_k-i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(4) $\sum_k |V_k|^2 n_k(m_1+1)\exp[-i(\omega_k-\omega_1)t]$

$$\times \frac{1}{(\omega_k-\omega_1-i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(5) $\sum_k |V_k|^2 \sqrt{n_1 m_1}(n_k+1)\exp[-i(\omega_1-\omega_k)t]$

$$\times \frac{1}{(\omega_1-\omega_k-i\epsilon)^2}\rho(n_1-1,n_k+1;m_1-1,n_k+1;0),$$

(6) $\sum_k |V_k|^2 n_k\sqrt{(n_1+1)(m_1+1)}\exp[-i(\omega_k-\omega_1)t]$

$$\times \frac{1}{(\omega_k-\omega_1-i\epsilon)^2}\rho(n_1+1,n_k-1;m_1+1,n_k-1;0),$$

(7) $\sum_k |V_k|^2 n_k\sqrt{(n_1+1)(m_1+1)}\exp[-i(\omega_1-\omega_k)t]$

$$\times \frac{1}{(\omega_1-\omega_k-i\epsilon)^2}\rho(n_1+1,n_k-1;m_1+1,n_k-1;0),$$

(8) $\sum_k |V_k|^2 \sqrt{n_1 m_1}(n_k+1)\exp[-i(\omega_k-\omega_1)t]$

$$\times \frac{1}{(\omega_k-\omega_1-i\epsilon)^2}\rho(n_1-1,n_k+1;m_1-1,n_k+1;0). \quad \text{(B3)}$$

Similarly, contributions from diagrams in Fig. 3 are

(1) $\sum_k |V_k|^2 (n_1+1)(n_k+1)\exp[-i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k-i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(2) $\sum_k |V_k|^2 n_k n_1 \exp[i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k+i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(3) $\sum_k |V_k|^2 (m_1+1)(n_k+1)\exp[i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k+i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(4) $\sum_k |V_k|^2 n_k m_1 \exp[-i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k-i\epsilon)^2}\rho(n_1,n_k;m_1,n_k;0),$$

(5) $\sum_k |V_k|^2 \sqrt{n_1 m_1}\, n_k \exp[-i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k-i\epsilon)^2}\rho(n_1-1,n_k-1;m_1-1,n_k-1;0),$$

(6) $\sum_k |V_k|^2 (n_k+1)\sqrt{(n_1+1)(m_1+1)}\exp[i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k+i\epsilon)^2}\rho(n_1+1,n_k+1;m_1+1,n_k+1;0),$$

(7) $\sum_k |V_k|^2 (n_k+1)\sqrt{(n_1+1)(m_1+1)}\exp[-i(\omega_1+\omega_k)t]$

$$\times \frac{1}{(\omega_1+\omega_k-i\epsilon)^2}\rho(n_1+1,n_k+1;m_1+1,n_k+1;0),$$

(8) $\sum_k |V_k|^2 \sqrt{n_1 m_1}\, n_k \exp[i(\omega_1+\omega_k)t]\frac{1}{(\omega_1+\omega_k+i\epsilon)^2}$

$$\times \rho(n_1-1,n_k-1;m_1-1,n_k-1;0). \quad \text{(B4)}$$

Adding all contributions (B3) and (B4), we obtain the following expression for $P^{(\nu_1)}\rho_{np}(t)$:

$$
\begin{aligned}
P^{(\nu_1)}\rho_{np}(t) \approx \lambda^2 \sum_k |V_k|^2 \Big\{ &[(n_k+1)(n_1+m_1)\rho(n_1,n_k;m_1,n_k;0) + n_k(n_1+m_1+2)\rho(n_1,n_k;m_1,n_k;0) \\
&+ (n_k+1)\sqrt{n_1 m_1}\,\rho(n_1-1,n_k+1;m_1-1,n_k+1;0) + n_k\sqrt{(n_1+1)(m_1+1)}\,\rho(n_1+1,n_k-1;m_1+1,n_k-1;0)] \\
&\times\left(\frac{e^{-i(\omega_k-\omega_1)t}}{(\omega_k-\omega_1-i\epsilon)^2}+\text{c.c.}\right) + [(n_k+1)(n_1+m_1+2)\rho(n_1,n_k;m_1,n_k;0) + n_k(n_1+m_1)\rho(n_1,n_k;m_1,n_k;0) \\
&+ n_k\sqrt{n_1 m_1}\,\rho(n_1-1,n_k-1;m_1-1,n_k-1;0) + (n_k+1)\sqrt{(n_1+1)(m_1+1)}\,\rho(n_1+1,n_k+1;m_1+1,n_k+1;0)] \\
&\times\left(\frac{e^{-i(\omega_k+\omega_1)t}}{(\omega_k+\omega_1-i\epsilon)^2}+\text{c.c.}\right)\Big\} e^{-i\nu_1\omega_1 t}.
\end{aligned} \tag{B5}
$$

In the thermodynamic limit $\Omega\to\infty$, Eq. (B5) leads to the desired expression (126) presented in the main text, where the factorization property (81) has been used.

APPENDIX C: THE $\lambda^2 T$ APPROXIMATION OF THE HU-PAZ-ZHANG KINETIC EQUATION

In this appendix we derive the $\lambda^2 t$ approximation of the Hu-Paz-Zhang kinetic equation given in Ref. [23]. Without any approximation, their kinetic equation is given by

$$
\begin{aligned}
i\hbar\frac{\partial}{\partial t}f(q_1,q_1',t) =& \left[-\frac{\hbar^2}{2M_1}\left(\frac{\partial^2}{\partial q_1^2}-\frac{\partial^2}{\partial q_1'^2}\right)+\frac{M_1\tilde{\Omega}_1^2(t)}{2}(q_1^2-q_1'^2)\right] \\
&\times f(q_1,q_1',t)-i\hbar\Gamma(t)(q_1-q_1') \\
&\times\left(\frac{\partial}{\partial q_1}-\frac{\partial}{\partial q_1'}\right)f(q_1,q_1',t)-i\Gamma(t)h(t) \\
&\times (q_1-q_1')^2 f(q_1,q_1',t)+\hbar\Gamma(t)f(t) \\
&\times (q_1-q_1')\left(\frac{\partial}{\partial q_1}+\frac{\partial}{\partial q_1'}\right)f(q_1,q_1',t),
\end{aligned} \tag{C1}
$$

where $\tilde{\Omega}_1$ is the renormalized frequency,

$$
\tilde{\Omega}_1^2(t)=\omega_1^2+\delta\Omega_1^2(t). \tag{C2}
$$

In the weak coupling approximation, the time dependent kernels appearing in the RHS of Eq. (C1) are given as [Eqs. (2.46a)–(2.46d) in [23]]

$$
\delta\Omega_1^2(t)\approx 2\lambda^2\int_0^t ds\,\eta(s)\cos(\omega_1 s),
$$

$$
\Gamma(t)\approx -\frac{\lambda^2}{\omega_1}\int_0^t ds\,\eta(s)\sin(\omega_1 s),
$$

$$
\Gamma(t)f(t)\approx \frac{\lambda^2}{\omega_1}\int_0^t ds\,\nu(s)\sin(\omega_1 s),
$$

$$
\Gamma(t)h(t)\approx \lambda^2\int_0^t ds\,\nu(s)\cos(\omega_1 s), \tag{C3}
$$

where we have explicitly written the coupling constant λ, and

$$
\nu(s)=\int_0^\infty d\omega J(\omega)\coth\left(\frac{\beta\hbar\omega}{2}\right)\cos(\omega s),
$$

$$
\eta(s)=-\int_0^\infty d\omega J(\omega)\sin(\omega s). \tag{C4}
$$

Here, the spectral density $J(\omega)$ is defined as

$$
J(\omega)=\sum_k \delta(\omega-\omega_k)\frac{C_k^2}{2m_k\omega_k} \tag{C5}
$$

and C_k is related to our v_k in Eq. (3) as

$$
v_k=\frac{C_k}{\sqrt{2m_k\omega_k}}. \tag{C6}
$$

To compare their kinetic equation with our equation (115) in our main text, we now evaluate Eq. (C3) in the $\lambda^2 t$ approximation. Using the second expression in Eq. (C4), we rewrite, for example, $\Gamma(t)$ as

$$
\begin{aligned}
\Gamma(t)\approx& \frac{\lambda^2}{\omega_1}\int_0^t ds\,\sin(\omega s)\int_0^\infty d\omega J(\omega)\sin(\omega s) \\
=& \lim_{\epsilon\to 0+}\frac{\lambda^2}{4\omega_1}\int_0^\infty d\omega J(\omega)\int_0^t ds(\exp[i(\omega_1+\omega+i\epsilon)s] \\
&+\exp[-i(\omega_1+\omega-i\epsilon)s]-\exp[i(\omega_1-\omega+i\epsilon)s] \\
&-\exp[-i(\omega_1-\omega-i\epsilon)s]).
\end{aligned} \tag{C7}
$$

In the large time limit, integration over s in the first exponential in Eq. (C7) yields

$$
\int_0^t ds\,\exp[i(\omega_1+\omega+i\epsilon)s]\to\frac{i}{\omega_1+\omega+i\epsilon}. \tag{C8}
$$

Performing integration for the remaining exponentials in Eq. (C7) and using the representation (A10), we obtain the expression for $\Gamma(t)$ in the large time limit as

$$\Gamma(t) \approx \frac{\pi\lambda^2}{2} \sum_k |V_k|^2 \delta(\omega_1 - \omega_k) = \frac{\lambda^2}{4\Omega} \sum_k \gamma_k = \lambda^2 \Gamma_2.$$
(C9)

Other kernels in Eq. (C3) are calculated in a similar way. Then, we obtain the following coefficients:

$$\delta\Omega_1^2(t) \approx 2\omega_1 \lambda^2 \delta\omega_1,$$

$$\Gamma(t)h(t) \approx \lambda^2 A_2,$$

$$\Gamma(t)f(t) \approx \lambda^2 D_{an} \equiv \lambda^2 P \int_0^\infty d\omega J(\omega) \coth\left(\frac{\beta\hbar\omega}{2}\right) \frac{1}{\omega_1^2 - \omega^2}.$$
(C10)

Coefficients Γ_2, $\delta\omega_1$, and A_2 are given in Eqs. (105), (118), and (116) of our main text. Replacing these coefficients by the expressions in the RHS of Eqs. (C9) and (C10) in Eq. (C1), we obtain the $\lambda^2 t$ approximation of their kinetic equation presented in Eq. (132) in our main text.

APPENDIX D: BOGOLIUBOV TRANSFORMATION

In this appendix we derive the exact solution for the reduced particle density matrix for zero temperature in the $\lambda^2 t$ limit (135).

By the Bogoliubov transformation presented below in Eq. (D1), the Hamiltonian (3) can be diagonalized in the continuous spectrum limit $\Omega \to \infty$ as in Eq. (29), where the new dressed field operator B_k is given by [40],

$$B_k = a_k + \omega_1 \lambda v_k G^-(\omega_k) \left[\int_{-\infty}^\infty dk' \lambda v_{k'} \left(\frac{a_{k'}}{\omega_{k'} - (\omega_k - i\epsilon)} \right. \right.$$
$$\left. \left. - \frac{a_{k'}^+}{\omega_{k'} + \omega_k} \right) - \frac{(\omega_k + \omega_1)a_1^+ + (\omega_k - \omega_1)a_1}{2\omega_1} \right].$$
(D1)

Here the function $G^\pm(\omega_k) \equiv G(\omega_k \pm i\epsilon)$ is defined as

$$G(z) \equiv \left[\omega_1^2 - z^2 - \int_{-\infty}^\infty dk' 2 \frac{\omega_1 \lambda^2 |v_{k'}|^2}{\omega_{k'}^2 - z^2} \right]^{-1}.$$
(D2)

Note that the diagonalized Hamiltonian (29) consists of the dressed field alone. The particle associated with the discrete spectrum has been destroyed by the resonance. This is the characteristic feature of the unstable system, which is nonintegrable in the sense of Petrosky and Prigogine [41]. The inverse transformation is given by

$$a_k = B_k - \omega_1 \lambda v_k \int_{-\infty}^\infty dk' \lambda v_{k'} \left[\frac{G^+(\omega_{k'})}{\omega_{k'} - \omega_k + i\epsilon} B_{k'} \right.$$
$$\left. - \frac{G^-(\omega_{k'})}{\omega_{k'} + \omega_k} B_{k'}^\dagger \right],$$
(D3)

$$a_1 = - \int_{-\infty}^\infty dk \lambda v_k [(\omega_k + \omega_1)G^+(\omega_k)B_k$$
$$- (\omega_k - \omega_1)G^-(\omega_k)B_k^\dagger].$$
(D4)

We now calculate the evolution of the density matrix $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ starting with the initial condition presented in Eq. (134). We note that the $\lambda^2 t$ limit has a well-defined meaning only in the context of the evolution of the density matrix, and not the wave function. Hence, in the following calculation this limit should be understood as applied in the context of the density matrix. Then, in the $\lambda^2 t$ limit, the Green function $G^-(\omega_k)$ approaches

$$G^-(\omega_k) \to \frac{1}{(z_+ - \omega_k)(z_+^* + \omega_k)} = \frac{1}{(z_+ - \omega_k)(\omega_1 + \omega_k)}$$
$$+ O(\lambda^2),$$
(D5)

where z_+ and z_+^* are the poles of $G^-(\omega_k)$ on the second Riemann sheet (the asterisk denotes complex conjugation). In the $\lambda^2 t$ limit, z_+ approaches

$$z_+ \to \omega_1 + \lambda^2 \delta\omega_1 + i\lambda^2 \frac{\gamma}{2}.$$
(D6)

The exact time evolution of the wave function of the total system is given by

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = e^{-iHt} a_1^+ |0_p, 0_k\rangle$$
$$= -e^{-iHt} \int_{-\infty}^\infty dk \lambda v_k [(\omega_k + \omega_1)G^-(\omega_k)B_k^\dagger$$
$$- (\omega_k - \omega_1)G^+(\omega_k)B_k]e^{\tilde{V}}|\tilde{0}\rangle.$$
(D7)

Here $|\tilde{0}\rangle$ denotes the true ground state, which is related to the bare ground state as

$$|0_p, 0_k\rangle = e^{\tilde{V}}|\tilde{0}\rangle,$$
(D8)

where $\tilde{V}$ is given as [40]

$$\tilde{V} = \frac{1}{2} \int_{-\infty}^\infty dk \int_{-\infty}^\infty dk' \lambda v_k \lambda v_{k'} \eta(-\omega_k + i\epsilon) \eta(-\omega_{k'}$$
$$+ i\epsilon)B_k^\dagger B_{k'}^\dagger \frac{\omega_1}{\omega_1 + \delta\epsilon_0} \left(1 - \frac{2(\omega_1 + \delta\epsilon_0)}{\omega_k + \omega_{k'}}\right)$$
(D9)

and $\delta\epsilon_0$ being the vacuum energy shift. The function $\eta(-z)$ is defined by

$$\eta(-z) \equiv \int_{-\infty}^\infty dk \frac{\omega_1 \lambda^2 |v_k|^2 |G(\omega_k)|^2}{\eta(\omega_k)(\omega_k - z)}$$
(D10)

and

$$\eta(z)\eta(-z) = G(z).$$
(D11)

In the $\lambda^2 t$ limit, the state $|1_p, 0_k\rangle$ in Eq. (133) can be connected to the state $|0_p, 1_k\rangle \equiv a_k^\dagger |0_p, 0_k\rangle$ with only one photon, and all other transitions vanish. Hence, in this limit, we have for the reduced density matrix of the particle

$$\hat{f}(t) = \text{Tr}_F[|\Psi(t)\rangle\langle\Psi(t)|] \rightarrow |1_p, 0_k\rangle |\langle 1_p, 0_k|\Psi(t)\rangle|^2 \langle 1_p, 0_k|$$
$$+ |0_p, 1_k\rangle |\langle 0_p, 1_k|\Psi(t)\rangle|^2 \langle 0_p, 1_k|. \qquad (D12)$$

In the position representation, Eq. (D12) leads in the $\lambda^2 t$ limit to

$$f(q_1, q_1', t) \rightarrow \phi_1(q_1) |\langle 1_p, 0_k|\Psi(t)\rangle|^2 \phi_1^*(q_1')$$
$$+ \phi_0(q_1) \sum_k |\langle 0_p, 1_k|\Psi(t)\rangle|^2 \phi_0^*(q_1'). \qquad (D13)$$

Now, using the Heisenberg representation of the dressed operator $B_k^\dagger$, i.e., $\exp[-iHt]B_k^\dagger\exp[iHt] = B_k^\dagger(0)\exp[-i\omega_k t]$, and the λ expansions as $|\tilde{0}\rangle = [1 + O(\lambda^2)]|0_p, 0_k\rangle$ and $B_k = a_k + O(\lambda)$, the Eq. (D7) leads in the $\lambda^2 t$ limit to

$$\langle 1_p, 0_k|\Psi(t)\rangle = \langle 1_p, 0_k|\lambda \int_{-\infty}^{\infty} dk \, v_k(\omega_k + \omega_1)$$
$$\times G^-(\omega_k) B_k^\dagger(0) e^{-i\omega_k t}|\tilde{0}\rangle$$
$$\rightarrow \lambda^2 \int_{-\infty}^{\infty} dk \, |v_k|^2 (\omega_1 + \omega_k)^2 |G^+(\omega_k)|^2 e^{-i\omega_k t}, \qquad (D14)$$

where we have used the fact that the true ground state is an invariant of motion, i.e., $\exp[-iHt]|\tilde{0}\rangle = |\tilde{0}\rangle$. Substituting Eq. (D4) into Eq. (D14) and performing the contour rotation of ω_k into the lower-half plane similar to Eqs. (129) and (130), we see that the contribution in the integration from the pure imaginary axis of ω_k vanishes in the $\lambda^2 t$ limit, while the pole at $\omega_k = z_1$ gives a finite contribution. Then, in the $\lambda^2 t$ limit, we obtain

$$\langle 1_p, 0_k|\Psi(t)\rangle \rightarrow 2\pi i\lambda^2 |v_{\omega_1 + \lambda^2\delta\omega_1 - i\lambda^2\gamma/2}|^2 \frac{1}{i\lambda^2\gamma}$$
$$\times \exp\left[-i\left(\omega_1 + \lambda^2\delta\omega_1 - i\lambda^2\frac{\gamma}{2}\right)t\right]$$
$$\rightarrow \exp\left[-i\left(\omega_1 + \lambda^2\delta\omega_1 - i\lambda^2\frac{\gamma}{2}\right)t\right]. \qquad (D15)$$

This leads to the final form of the exact expression for $|\langle 1_p, 0_k|\Psi(t)\rangle|^2$ in the $\lambda^2 t$ limit as

$$|\langle 1_p, 0_k|\Psi(t)\rangle|^2 \rightarrow e^{-\lambda^2\gamma t}. \qquad (D16)$$

Calculation of the coefficient $|\langle 0_p, 1_k|\Psi(t)\rangle|^2$ is similar to the one presented above and in the $\lambda^2 t$ limit, we obtain

$$\sum_k |\langle 0_p, 1_k|\Psi(t)\rangle|^2 \rightarrow 1 - e^{-\lambda^2\gamma t}. \qquad (D17)$$

Substituting Eqs. (D16) and (D17) into Eq. (D13), we obtain the exact solution of $f(q, q', t)$ in the $\lambda^2 t$ limit given by Eq. (136) in the main text.

[1] D. Driebe, *Fully Chaotic Maps and Broken Time Symmetry* (Kluwer Academic Publishers, Dordrecht, 1999).

[2] E. C. G. Sudarshan, C. Chiu, and V. Gorini, Phys. Rev. D **18**, 2914 (1978).

[3] A. Böhm and M. Gadella, *The Rigged Hilbert Space and Quantum Mechanics*, Springer Lecture Notes on Physics Vol. 78 (Springer, New York, 1978).

[4] T. Petrosky, I. Prigogine, and S. Tasaki, Physica (Amsterdam) A **173**, 175 (1991).

[5] T. Petrosky and I. Prigogine, Chaos, Solitons Fractals **7**, 441 (1996).

[6] T. Petrosky and I. Prigogine, Adv. Chem. Phys. **99**, 1 (1997).

[7] T. Petrosky and I. Prigogine, in *Gravity, Particles and Space-time*, edited by P. Pronin and G. Sardanashvily (World Scientific, Singapore, 1996).

[8] W. G. Unruh and W. H. Zurek, Phys. Rev. D **40**, 1071 (1989).

[9] D. Giulini *et al.*, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer-Verlag, New York, 1996).

[10] A. Shimony, in *The New Physics*, edited by P. Davies (Cambridge University Press, New York, 1993).

[11] N. Bohr and L. Rosenfeld, Phys. Rev. **78**, 794 (1950).

[12] A. Böhm and M. Gadella, *Dirac Kets, Gamov Vectors and Gelfand Triplets*, Springer Lecture Notes on Physics Vol. 348 (Springer, New York, 1989).

[13] J. Kumicak and E. Brändas, Int. J. Quantum Chem. **32**, 669 (1987).

[14] C. George, Physica (Amsterdam) **65**, 277 (1973).

[15] C. George, F. Mayné, and I. Prigogine, Adv. Chem. Phys. **61**, 223 (1985).

[16] T. Petrosky and H. Hasegawa, Physica (Amsterdam) A **160**, 351 (1989).

[17] I. Prigogine, *Nonequilibrium Statistical Mechanics* (Wiley, New York, 1962).

[18] I. Prigogine and P. Résibois, Physica (Amsterdam) **27**, 629 (1974).

[19] J. P. Boon and S. Yip, *Molecular Hydrodynamics* (McGraw-Hill, New York, 1980).

[20] T. Petrosky, Found. Phys. **29**, 1417 (1999).

[21] T. Petrosky, Found. Phys. **29**, 1581 (1999).

[22] A. O. Caldeira and A. J. Leggett, Physica A **121**, 587 (1983).

[23] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992).

[24] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **47**, 1576 (1993).

[25] L. Tessieri, D. Vitali, and P. Grigolini, Phys. Rev. A **41**, 4404 (1995).

[26] E. C. G. Sudarshan and B. Misra, J. Math. Phys. **18**, 756 (1977).

[27] C. Compgano, R. Passante, and F. Persico, *Atom-Field Interactions and Dressed Atoms* (Cambridge University Press, Cambridge, 1995).

[28] T. Petrosky and V. Barsegov (unpublished).

[29] T. Petrosky and V. Barsegov, *in Advanced Studies in Astrophysics and Cosmology: The Chaotic Universe*, edited by V. Gurzadyan and R. Ruffini (World Scientific, Singapore, 1999).

[30] A. Kossakowski, Rep. Math. Phys. **3**, 247 (1972).

[31] A. Kossakowski, Bull. Acad. Pol. Sci., Ser. Sci., Math., Astron. Phys. **20**, 1021 (1972).

[32] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. **17**, 821 (1976).

[33] P. Facchi and S. Pascazio, Physica (Amsterdam) A **271**, 133 (1999).

[34] E. Joos and H. D. Zeh, Z. Phys. B: Condens. Matter **59**, 223 (1985).

[35] M. Tegmark, Found. Phys. Lett. **6**, 571 (1993).

[36] K. O. Friedrichs, Commun. Pure Appl. Math. **1**, 361 (1948).

[37] K. O. Friedrichs, *Mathematical Aspects of the Quantum Theory of Fields* (Interscience, New York, 1953).

[38] E. C. G. Sudarshan, *Relativistic Particle Interactions*, Lecture Notes at University of Rochester, 1962.

[39] G. Bhamati and E. C. G. Sudarshan, Int. J. Mod. Phys. B **10**, 1531 (1996).

[40] E. Karpov, I. Progogine, T. Petrosky, and G. Pronko, J. Math. Phys. **41**, 118 (2000).

[41] T. Petrosky and I. Prigogine, Physica (Amsterdam) A **147**, 461 (1988).